HIGHER ALGEBRA REVISION

(Predicted question types)

Q1.

Expand and simplify $(y + 5)(y - 4)$

_____

_____

_____

Answer _____

(Total 2 marks)

Q2.

Expand and simplify $(2x + 5y)(3x - 8y)$

_____

_____

_____

Answer _____

(Total 3 marks)

Q3. Solve $x^2 = 30.25$

_____

Answer _____

(Total 2 marks)

Q4. $2x^2 - 20x + c \equiv a(x - b)^2 + 3b$

Work out the value of c.

_____

_____

_____

_____

_____

Answer _____

(Total 3 marks)

Q5. For all values of x, $f(x) = x^2 + 1$ $g(x) = x - 5$

(a) Show that $fg(x) = x^2 - 10x + 26$

(2)

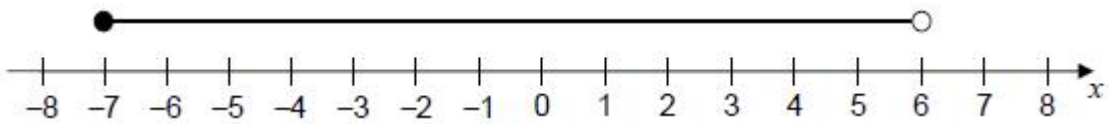
(b) Solve $fg(x) = gf(x)$

$x =$ _____

(4)

(Total 6 marks)

Q6. Circle the inequality shown by the diagram.



$-7 < x < 6$ $-7 \leq x < 6$ $-7 < x \leq 6$ $-7 \leq x \leq 6$ **(Total 1 mark)**

Q7. Circle the equation of the x-axis.

$x + y = 0$ $x - y = 0$ $x = 0$ $y = 0$

(Total 1 mark)

Q8. Simplify fully $(8x^3 y^5)^2$

_____

_____

Answer _____

(Total 2 marks)

Q9.

$x(x + 4) = x^2 + 4x$

For how many values of x is $x(x + 4)$ equal to $x^2 + 4x$
Circle your answer.

0 1 2 all

(Total 1 mark)

Q10.

At a concert
3 adult and 4 child tickets cost £23
1 adult and 5 child tickets cost £15

Work out the cost of an adult ticket and the cost of a child ticket.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Cost of an adult ticket £ .....

Cost of a child ticket £ .....

(Total 4 marks)

Q11.

(a) Factorise fully $9a^2 - 6a$

_____

_____

Answer _____

(2)

(b) Solve $x^2 - 12x + 20 = 0$

_____

_____

Answer _____

(3)

(Total 5 marks)

Q12.

Rearrange $c = \frac{4 - d}{d + 3}$ to make d the subject.

_____

_____

_____

_____

_____

_____

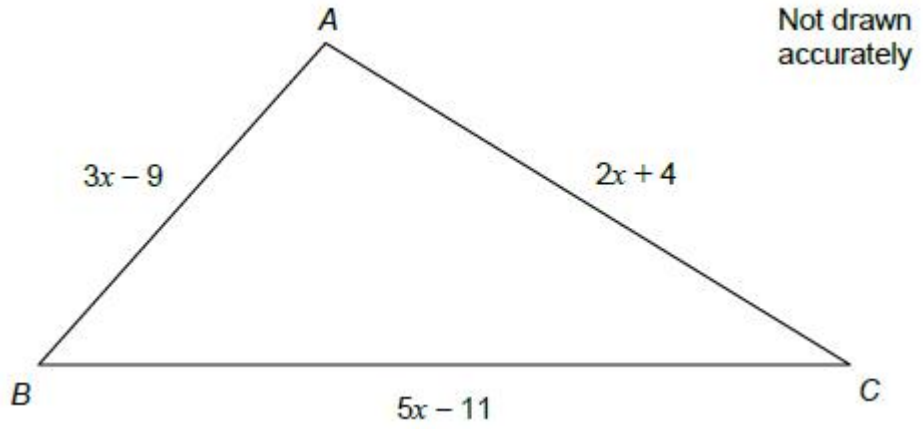
_____

Answer _____

(Total 4 marks)

Q13.

In this question all lengths are in centimetres.



Given $AB : BC = 1 : 2$

show that $AC : BC = 3 : 4$

(Total 5 marks)

Q14. Use the quadratic formula to solve $5x^2 + 11x - 2 = 0$

Give your solutions to 2 decimal places.

Answer _____

(Total 3 marks)

Q15. Work out the equation of the line that

is parallel to the line $y = 5x - 3$

passes through $(-2, -4)$

Answer _____

(Total 3 marks)

Q16. (a) Factorise $x^2 - y^2$

Answer _____

(1)

(b) Solve $\frac{2x}{5} + 1 = 13$

$x =$ _____

(3)

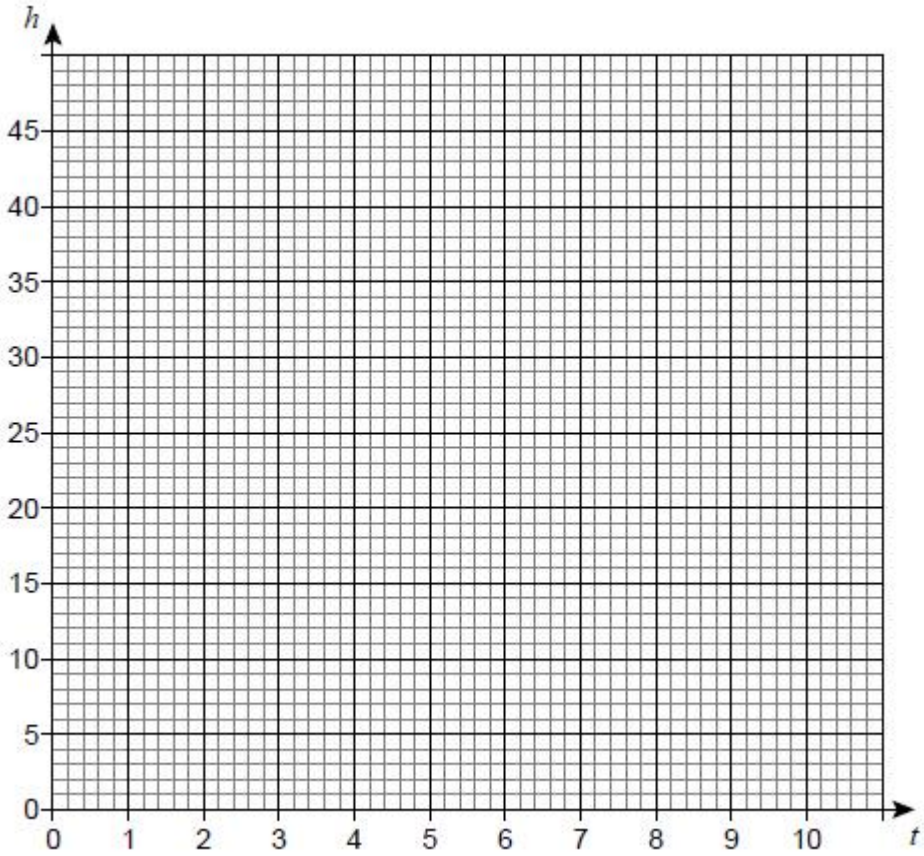
(Total 4 marks)

Q17. The height, h metres, of a particle at time, t seconds, is given by the function

$h = 0 \qquad 0 \le t < 2$

$h = (14 - t)(t - 2) \qquad 2 \le t \le 10$

(a) Draw a graph to show the height of the particle in the first 10 seconds.



(3)

(b) By joining the points on the graph where $t = 3$ and $t = 7$ with a straight line, work out the average rate of change of height between 3 and 7 seconds.

Answer ..... m/s

(2)

(Total 5 marks)

Q18.

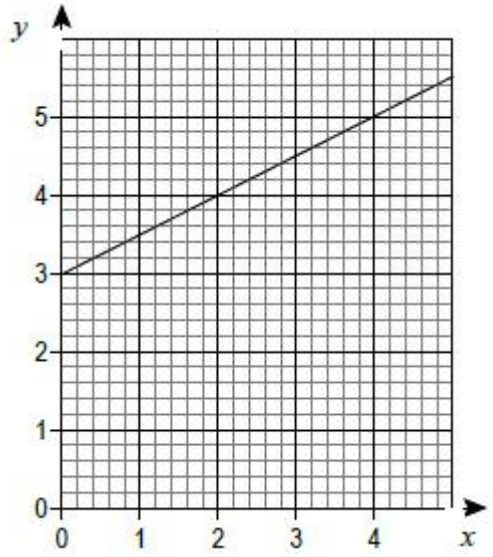
Show that $\dfrac{2w+4}{w^2-25} \times \dfrac{w+5}{w^2+3w+2} \times (3w^2-16w+5)$

simplifies to $\dfrac{aw+b}{cw+d}$ where a, b, c and d are integers.

(Total 5 marks)

Q19.

Here is the graph of a straight line.



(a) Work out the equation of the line.

_____

_____

_____

Answer _____

(2)

(b) Work out the coordinates of the y-intercept of the line that

is parallel to the line in part (a)
and passes through (4, 1)

_____

_____

Answer (..... , ...)

(2)

(Total 4 marks)

Q20.

$f(x) = 8x - 5$

Circle the value of f(−2)

11 −21 −11 1

(Total 1 mark)

Q21.

Use the iteration $x_{n+1} = 4 - \frac{2}{x_n^2}$

to work out an approximate solution to $x = 4 - \frac{2}{x^2}$

Start with $x_1 = 1$
Give your answer to 2 decimal places.

Answer _____

(Total 3 marks)

Q22.

Kris is simplifying $3n^5 \times 2n^4$
His answer is $5n^{20}$

Identify any mistakes he has made.

(Total 2 marks)

Q23.

Prove that $5n - (2n + 3)(n + 1)$ is always negative.

(Total 3 marks)

Q24.

$f(x) = 3^{2x}$ and $g(x) = x^3$ for all values of x.

(a) Work out the value of $f(1) + g(4)$

Answer _____

(2)

(b) Work out the value of $g^{-1}(-27)$

Answer _____

(2)

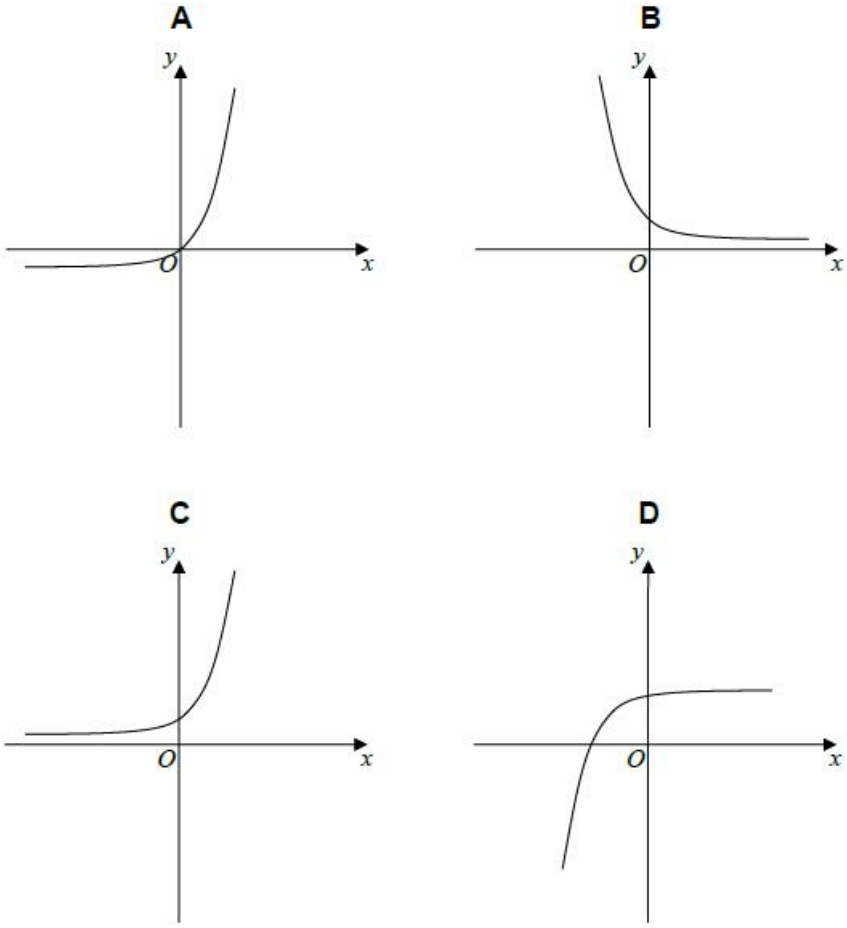
(c) Work out an expression for $gf(x)$

Give your answer as a power of 3 in its simplest form.

Answer _____

(2)

(d) One of these graphs is a sketch of $y = 3^{2x}$
Which one?
Circle the correct letter.



(1)
(Total 7 marks)

Q25.(a) Write $x^2 + 6x + 10$ in the form $(x + a)^2 + b$

_____

_____

_____

_____

_____

Answer _____

(2)

(b) Hence, write down the coordinates of the turning point of the curve $y = x^2 + 6x + 10$

Answer (..... , ...)

(1)
(Total 3 marks)

Q26.

I am thinking of two numbers.
If I multiply the first number by 4 and add the second number the answer is 32
If I multiply the first number by 2 and add the second number the answer is 23

Work out the two numbers.

First number .....

Second number .....

(Total 4 marks)

Q27.

Circle the identity.

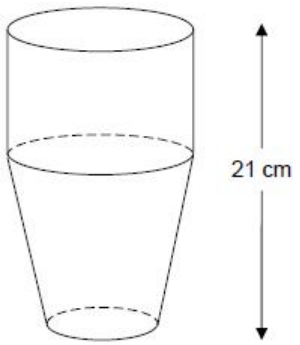
$(x - 3)^2$ $\qquad$ $(x - 3)^2 > 5$

$(x - 3)^2 = 1 - 6x$ $\qquad$ $(x - 3)^2 \equiv x^2 - 6x + 9$

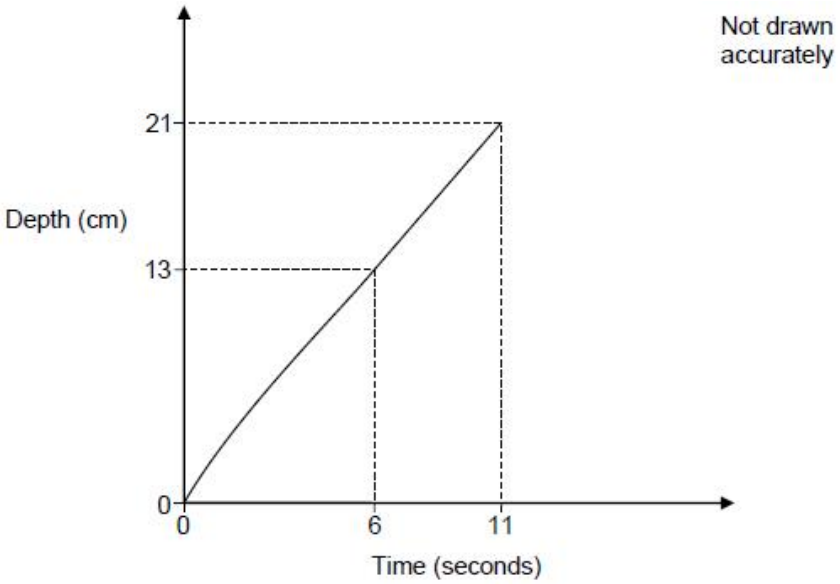
(Total 1 mark)

Q28.

The diagram shows an empty container of height 21 cm
The container consists of a cylinder on a frustum of a cone.



Water is added to the container at a constant rate for 11 seconds.
The sketch graph shows the depth of the water as the container fills.
The graph is a curve for the first 6 seconds and a straight line for the next 5 seconds.



(a) Circle the height of the cylinder.

8 cm 10.5 cm 13 cm 21 cm

(1)

(b) Work out the rate of increase of the depth of the water between 6 seconds and 11 seconds.
State the units of your answer.

_____

_____

_____

Answer _____

(3)

(Total 4 marks)

Q29. Solve the equation $\frac{6}{x+3} + \frac{1}{2x+5} = 3$

Give your answers to 2 decimal places.

Answer _____ and _____

(Total 6 marks)

Q30.

Solve the simultaneous equations

$3x + 2y = 10$

$3x - y = 13$

Answer x = ... y = ...

(Total 3 marks)

Q31.

Work out an approximate solution to $x^3 + 3x - 1 = 0$

Use the iteration $x_{n+1} = \dfrac{1}{x_n^2 + 3}$

Start with $x_1 = 1$

Give your answer to 2 decimal places.

_____

_____

_____

_____

_____

_____

Answer _____

(Total 3 marks)

Q32.

n is an odd number.

Tick any statements that are true.

☐	$n^2 + 1$	*is odd*
☐	$n(n + 1)$	*is odd*
☐	$(n + 1)^2$	*is odd*
☐	$n^2 + 2$	*is odd*
☐	$n(n + 2)$	*is odd*
☐	$(n + 2)^2$	*is odd*

(Total 3 marks)

Q33.

Prove that $3(x + 1)(x + 7) - (2x + 5)^2$ *is never positive.*

(Total 5 marks)

Q34. *For all values of x,* $f(x) = \frac{4x - 3}{5}$

Work out $f^{-1}(x)$

Answer _____

(Total 3 marks)

Q35.

w, x and y are three integers.

w is 2 less than x
y is 2 more than x

Prove that $wy + 4 = x^2$

(Total 3 marks)

Mark schemes

Q1.

$y^2 - 4y + 5y - 20$

Allow 1 error

M1

$y^2 + y - 20$

A1

[2]

Q2.

$6x^2 - 16xy + 15xy - 40y^2$

Allow one error

M1

$6x^2 - 16xy + 15xy - 40y^2$

Fully correct

A1

$6x^2 - xy - 40y^2$

ft their four terms

A1ft

Q3.

5.5 and −5.5

oe

B1 for each

B2

Additional Guidance

± 5.5

B2

[2]

Q4.

Alternative method 1

$ax^2 - abx - abx + ab^2 + 3b$

M1

$a = 2$ and $b = 5$

A1

65

A1

Alternative method 2

$2[(x - 5)^2 - 50 + c]$

M1

$b = 5$

M1

65

A1

[3]

Q5.

(a) $(x - 5)^2 + 1$

M1

$x^2 - 5x - 5x + 25 + 1$
$= x^2 - 10x + 26$

A1

(b) $x^2 + 1 - 5$ or $x^2 - 4$

B1

$x^2 - 10x + 26 =$ their $(x^2 - 4)$

M1

$-10x = -4 - 26$
or $-10x = -30$
or $10x = 30$
oe

M1

3

A1

[6]

Q6.

$-7 \le x < 6$

B1

[1]

Q7.

$y = 0$

B1

[1]

Q8.

$64x^6y^{10}$

B1 for two terms correct

B2

[2]

Q9.

all

B1

[1]

Q10.

Alternative method 1

3a (+) 4c (=) 23
and
3a (+) 15c (=) 45
or
15a (+) 20c (=) 115
and
4a (+) 20c (=) 60

oe eg works in pence
Multiplies one or both equation(s) to equate coefficients of a or c
Allow one error in multiplication

M1

11c (=) 22
or
11a (=) 55

oe
Subtracts equations to eliminate one variable
Allow one error in subtraction

M1

(a =) 5 or (c =) 2

A1

(a =) 5 and (c =) 2

A1

Alternative method 2

$a = \frac{23 - 4c}{3}$
or a = 15 − 5c
or
$c = \frac{23 - 3a}{4}$
or $c = \frac{15 - a}{5}$

oe
Makes a or c the subject

M1

$\frac{23 - 4c}{3} = 15 - 5c$
or
$\frac{23 - 3a}{4} = \frac{15 - a}{5}$

oe

Correctly substitutes their expression to eliminate one variable

M1

(a =) 5 or (c =)2

A1

(a =) 5 and (c =) 2

A1

Additional Guidance

Accept any letters, or 'adult' and 'child', as variables
To allow one error in the first mark of alt 1, the 'equal' coefficients must be the same. eg
allow 3a + 4c = 23 and 3a + 15c = 15
but not 3a + 4c = 23 and 3a + 5c = 45

[4]

Q11.

(a) $3a(3a - 2)$

B1 $a(9a - 6)$ *or* $3(3a^2 - 2a)$

B2

(b) $(x + a)(x + b)$

where $ab = 20$

or $a + b = -12$

M1

$(x - 2)(x - 10)$

A1

2 and 10

ft their pair of brackets

B1ft

[5]

Q12.

$c(d + 3) = 4 - d$

M1

$cd + 3c = 4 - d$

M1dep

$cd + d = 4 - 3c$
or $d(c + 1) = 4 - 3c$

M1dep

$d = \frac{4 - 3c}{c + 1}$

oe $d = \frac{-4 + 3c}{-c - 1}$

A1

[4]

Q13.

$2(3x - 9) = 5x - 11$

oe Must be correct and have x on both sides

M1

$6x - 18 = 5x - 11$

oe their bracket(s) expanded correctly

M1

$x = 7$

A1

$2 \times$ their $7 + 4$ or 18
and
$5 \times$ their $7 - 11$ or 24

Substitutes their value of x in

$2x + 4$ and $5x - 11$

M1

$18 : 24$ and $3 : 4$

oe eg $\frac{18}{24}$ and $\frac{3}{4}$

A1

Additional Guidance

$3x - 9 = 2(5x - 11)$

M0

$3x - 9 = 10x - 22$

M1

$x = \frac{13}{7}$

A0

$2 \times \frac{13}{7} + 4$ and $5 \times \frac{13}{7} - 11$

M1 A0

T & I leading to $x = 7$

M1 M1 A1

[5]

Q14.

$$\frac{-11 \pm \sqrt{11^2 - 4 \times 5 \times (-2)}}{2 \times 5}$$

Allow one error

Condone missing brackets

M1

$$\frac{-11 \pm \sqrt{11^2 - 4 \times 5 \times (-2)}}{2 \times 5}$$

or

$$\frac{-11 \pm \sqrt{161}}{10}$$

or
−2.3688…. and 0.1688…
or
−2.37
or
0.17

oe
Fully correct
Condone missing brackets

A1

−2.37 and 0.17

A1

Additional Guidance

Condone the method of completing the square for M1A1A0 or M1A1A1

[3]

Q15.

Gradient is 5

Implied by y = 5x...

B1

their 5 × −2 + c = −4

M1

y = 5x + 6 or 5x − y + 6 = 0
or y − 5x − 6 = 0

ft their gradient

A1 ft

[3]

Q16.

(a) *(x – y)(x + y)*

B1

(b) $\frac{2x}{5}$ *= 13 − 1 or* $\frac{2x}{5}$ *= 12*

(13 − 1) × 5 scores M1

M1

2x = their 12 × 5
or
2x = their 65 − their 5 or 2x = 60

oe

(13 − 1) × 5 ÷ 2 scores M1M1

M1

30

A1

Additional Guidance

Embedded answer

eg $\frac{2 \times 30}{5} + 1 = 13$

M1M1A0

eg $\frac{60}{5} + 1 = 13$

M1M0A0

[4]

Q17.

(a) Smooth curve passing through the points (± 0.5 square)
(2, 0), (3, 11), (4, 20), (5, 27), (6, 32), (7, 35), (8, 36), (9, 35), (10, 32)

Accept a line drawn from (0,0) to (2,0)

Condone no line drawn from (0,0) to (2,0))

B2 At least 7 correct points worked out or plotted (± 0.5 square)

B1 At least 4 correct points worked out or plotted (± 0.5 square)

B3

Additional Guidance

Correct points may be implied by curve passing through the points
Condone curve continued beyond t = 10

(b) $\frac{\text{their } 35 - \text{their } 11}{7 - 3}$ or $\frac{24}{4}$

M1

6

ft their points

A1ft

[5]

Q18.

$(w + 5)(w - 5)$

B1

$(w + 1)(w + 2)$

B1

$(3w + a)(w + b)$

$ab = 5$ or $a + 3b = -16$

M1

$(3w - 1)(w - 5)$

A1

$\frac{6w-2}{w+1}$

A1

[5]

Q19.

(a) $y = \frac{1}{2}x + 3$

B1 for $\frac{1}{2}x + c$ or $mx + 3$

or gradient = $\frac{1}{2}$ oe

B2

(b) (0, −1)

B1 for each coordinate or for reverse coordinate or $y = -1$ seen

or for $y = \frac{1}{2}x + c$

or gradient = $\frac{1}{2}$

B2

[4]

Q20.

−21

B1

[1]

Q21.

$x_2 = 2$ and $x_3 = 3.5$

M1

$x_4 = 3.83(6...)$ and $x_5 = 3.86(4...)$

M1

3.87

A1

[3]

Q22.

Any valid statement about the coefficient

e.g. 5 should be 6

he has added 3 and 2 (instead of multiplying)

he should have multiplied 3 and 2

B1

Any valid statement about the power

e.g. 20 should be 9
he has multiplied 5 and 4 (instead of adding)
he should have added 5 and 4

B1

Additional Guidance

$6n^9$ identified as the correct answer

B1B1

It should be 6 and 9

B1

It should be $6n^{20}$

B1

It should be $5n^9$

B1

It should be 6

B0

It should be 9

B0

[2]

Q23.

Alternative method 1

$(5n -)\ 2n^2 + 2n + 3n + 3$
or $(5n -)\ 2n^2 + 5n + 3$
oe
Allow one error

M1

$-2n^2 - 3$

A1

$-2n^2 - 3$ and $n^2 \geq 0$ so $-2n^2 \leq 0$
and subtracting 3 means always negative
oe valid argument
eg $-2n^2 - 3$ and $-2n^2 - 3 \leq -3$ because $n^2 \geq 0$ and $-2n^2 \leq 0$

A1

Alternative method 2

$(5n -)\ 2n^2 + 2n + 3n + 3$
or $(5n -)\ 2n^2 + 5n + 3$
oe
Allow one error

M1

$-(2n^2 + 3)$

A1

$-(2n^2 + 3)$ and $n^2 \geq 0$ so $2n^2 \geq 0$

and adding 3 means $2n^2 + 3$ always positive

oe valid argument

eg $-(2n^2 + 3)$ and $2n^2 + 3 \geq 3$ because $n^2 \geq 0$ and $-2n^2 \geq 0$

A1

Additional Guidance

M1 One error is implied by expansion of $(2n + 3)(n + 1)$ to either
$2n^2 + 5n + ...$ or $... + 5n + 3$
Expansion without working seen of $(2n + 3)(n + 1)$ to
$2n^2 + kn + 3$ where $k \neq 5$

M0

[3]

Q24.

(a) *9 or 64 or $3^2 + 4^3$*

M1

73

A1

(b) $\sqrt[3]{x}$ *or* $\sqrt[3]{-27}$ *or $x^3 = -27$*

M1

−3

A1

(c) $(3^{2x})^3$

M1

3^6x

A1

(d) *C*

B1

[7]

Q25.

(a) ***Alternative method 1***

$(x + 3)^2 + \ldots$ *or* $a = 3$

M1

$(x + 3)^2 + 1$

Accept $a = 3$ and $b = 1$

A1

Alternative method 2

$2a = 6$ *and* $a^2 + b = 10$

M1

$(x + 3)^2 + 1$

Accept $a = 3$ and $b = 1$

A1

(b) *(−3, 1)*

oe

ft their a and their b

B1ft

[3]

Q26.

Alternative method 1

$4x + y = 32$
and $2x + y = 23$

oe

using any letters or words

M1

$4x - 2x = 32 - 23$
or $2x = 9$
or $2y - y = 46 - 32$

oe

elimination of a letter

M1

First number = 4.5
and second number = 14

oe

A1

First number = 4.5
and second number = 14

oe

SC3 First number = 14

and second number = 4.5

A1

Alternative method 2

Identifies a pair of values that satisfy one statement and correctly evaluates the second statement for those values

M1

Identifies a different pair of values that satisfy one statement and correctly evaluates the second statement for those values

M1

First number = 4.5
and second number = 14

oe

SC3 First number = 14

and second number = 4.5

A2

Additional Guidance

A1 is not possible in alternative method 2

[4]

Q27.

$(x - 3)^2 \equiv x^2 - 6x + 9$

B1

[1]

Q28.

(a) 8 cm

B1

(b) $\frac{21-13}{11-6}$

oe

M1

$\frac{8}{5}$ or 1.6

oe

A1

cm/s or cm s^{-1}

oe eg Centimetres per second

B1

[4]

Q29.

$6(2x + 5) + 1(x + 3)$ or $3(x + 3)(2x + 5)$

oe

May be seen as part of a fraction or fractions with denominator $(x + 3)(2x + 5)$

M1

$6(2x + 5) + 1(x + 3) = 3(x + 3)(2x + 5)$

oe

M1dep

$6x^2 + 20x + 12 (= 0)$

or

$3x^2 + 10x + 6 (= 0)$

Simplifying the expression to three terms

A1

$$\frac{-20 \pm \sqrt{20^2 - 4 \times 6 \times 12}}{2 \times 6}$$

or $$\frac{-10 \pm \sqrt{10^2 - 4 \times 3 \times 6}}{2 \times 3}$$

oe

Allow one error

–2.548... or –0.784...

Strictly ft their quadratic

M1

$$\frac{-20 \pm \sqrt{20^2 - 4 \times 6 \times 12}}{2 \times 6}$$

or $$\frac{-10 \pm \sqrt{10^2 - 4 \times 3 \times 6}}{2 \times 3}$$

oe

fully correct

A1ft

–0.78 and –2.55

A1

Additional Guidance

One correct solution to 2 or more dp implies 4 marks

Two correct solutions to more than 2 dp implies 5 marks

$3x^2 + 10x = -6$

M1M1A1

ft their quadratic for the 4th and 5th marks

If no real roots M1A1ft can still be awarded

If quadratic factorises, must see correct factors for M1 and correct solutions for A1ft

If quadratic does not factorise, attempt to factorise scores M0

"Their quadratic" must be in the form $ax^2 + bx + c$ (= 0) or equivalent, no credit for solving a quadratic embedded within fractions etc

[6]

Q30.

$2y - -y = 10 - 13$

or $3y = -3$

or

$3x + 6x = 10 + 26$

or $9x = 36$

Eliminates a variable

M1

$y = -1$ or $x = 4$

A1

$y = -1$ and $x = 4$

A1

[3]

Q31.

$x2 = 0.25$

oe

M1

0.3218… or 0.3222…

oe

A1

0.32

ft their 3 dp value or better

B1ft

[3]

Q32.

Last three boxes ticked

B2 for three correct and one incorrect
or two correct and one incorrect
or two correct and none incorrect
B1 for three correct and two incorrect
or two correct and two incorrect
or one correct and none incorrect

B3

[3]

Q33.

$x^2 + x + 7x + 7$
or $x^2 + 8x + 7$

oe

M1

$3x^2 + 24x + 21$

M1dep

$4x^2 + 10x + 10x + 25$
or $4x^2 + 20x + 25$

oe

M1

$-x^2 + 4x - 4$

oe

A1

$-(x - 2)^2$ *so never positive*

A1

[5]

Q34.

$5f(x) = 4x - 3$ or $5f(x) + 3 = 4x$
or $5y = 4x - 3$ or $5y + 3 = 4x$
or $5x = 4y - 3$ or $5x + 3 = 4y$

Accept any letter used for y

M1

$\frac{5f(x)+3}{4}$ *(= x)*

or $\frac{5y+3}{4}$ *(= x)*

M1

$\frac{5x+3}{4}$

Condone y = (or any other letter)

A1

[3]

Q35.

Alternative method 1

(w =) x − 2 and (y =) x + 2

Allow (x =) w + 2 and (x =) y − 2

M1

(x − 2)(x + 2) + 4
or
wy = (x − 2)(x + 2) and wy = $x^2 - 4$

M1

$= x^2 - 4 + 4$
and $x^2 - 4 + 4 = x^2$

All steps must be seen
SC1 correct numerical example with all steps shown

A1

Alternative method 2

(x =) w + 2 and (y =) w + 4

Allow (x=) w + 2 and (x =) y − 2

M1

(w)(w + 4) + 4

M1

$= w^2 + 4w + 4$
and $w^2 + 4w + 4 = (w + 2)^2$
and $(w + 2)^2 = x^2$

All steps must be seen
SC1 correct numerical example with all steps shown

A1

Alternative method 3

(x =) y − 2 and (w =) y − 4

Allow (x =) w + 2 and (x =) y − 2

M1

(y)(y − 4) + 4

M1

$= y^2 - 4y + 4$
and $y^2 - 4y + 4 = (y - 2)^2$
and $(y - 2)^2 = x^2$

All steps must be seen
SC1 correct numerical example with all steps shown

A1

Additional Guidance

$x = 3$, $w = 1$, $y = 5$ and $1 \times 5 + 4 = 9$

0

$x = 3$, $w = 1$, $y = 5$ and $1 \times 5 + 4 = 9$ and $9 = 3^2$

SC1

$1 \times 5 + 4 = 9$ and $9 = 3^2$

0

[3]