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Recurring Decimals

1) a) Convert the recurring decimal $0.\dot{3}\dot{6}$ to a fraction in its simplest form.

   b) Prove that the recurring decimal $0.\dot{7}\dot{2} = \frac{8}{11}$

2) a) Change $\frac{4}{9}$ to a decimal.

   b) Prove that the recurring decimal $0.\dot{5}\dot{7} = \frac{19}{33}$

3) a) Change $\frac{3}{11}$ to a decimal.

   b) Prove that the recurring decimal $0.\dot{4}\dot{5} = \frac{15}{33}$

4) a) Change $\frac{1}{6}$ to a decimal.

   b) Prove that the recurring decimal $0.\dot{1}3\dot{5} = \frac{5}{37}$

5) a) Convert the recurring decimal $0.\dot{2}6\dot{1}$ to a fraction in its simplest form.

   b) Prove that the recurring decimal $0.2\dot{7} = \frac{5}{18}$

6) a) Convert the recurring decimal $5.\dot{2}$ to a fraction in its simplest form.

   b) Prove that the recurring decimal $0.1\dot{3}\dot{6} = \frac{3}{22}$

Fractional and Negative Indices

1) Simplify the following:

a) $y^4 \times y^5$

b) $x^2 \times x^6$

c) $(p^4)^5$

d) $(x^3)^2$

e) $(x^4)^{-2}$

f) $(x^{-3})^{-5}$

g) $x^7 \div x^2$

h) $\dfrac{t^5}{t^3}$

2) Work out the value of the following, leaving your answer in fraction form when necessary

a) 5^0

b) 4^{-2}

c) 5^{-3}

d) $49^{\frac{1}{2}}$

e) $8^{\frac{1}{3}}$

f) $32^{\frac{2}{5}}$

g) $16^{-\frac{1}{2}}$

h) $27^{-\frac{1}{3}}$

i) $64^{-\frac{2}{3}}$

3) $5\sqrt{5}$ can be written in the form 5^n.

Calculate the value of n.

4) $2\sqrt{8}$ can be written in the form 2^n.

Calculate the value of n.

5) $a = 2^x,\ b = 2^y$

Express in terms of a and b

(i) 2^{x+y}

(ii) 2^{2x}

(iii) 2^{x+2y}

Surds

1) Simplify the following:

a) $\sqrt{7} \times \sqrt{7}$

b) $\sqrt{3} \times \sqrt{3}$

c) $\sqrt{20}$

d) $\sqrt{24}$

e) $\sqrt{72}$

f) $\sqrt{200}$

g) $\sqrt{\frac{2}{25}}$

2) Simplify the following:

a) $\sqrt{2} \times \sqrt{18}$

b) $\sqrt{8} \times \sqrt{32}$

c) $\sqrt{99} \times \sqrt{22}$

d) $\sqrt{45} \times \sqrt{20}$

e) $\sqrt{18} \times \sqrt{128}$

f) $\sqrt{28} \times \sqrt{175}$

3) Expand and simplify where possible:

a) $\sqrt{3}(3-\sqrt{3})$

b) $\sqrt{2}(6+2\sqrt{2})$

c) $\sqrt{7}(2+3\sqrt{7})$

d) $\sqrt{2}(\sqrt{32}-\sqrt{8})$

4) Expand and simplify where possible:

a) $(1+\sqrt{2})(1-\sqrt{2})$

b) $(3+\sqrt{5})(2-\sqrt{5})$

c) $(\sqrt{3}+2)(\sqrt{3}+4)$

d) $(\sqrt{5}-3)(\sqrt{5}+1)$

e) $(2+\sqrt{7})(2-\sqrt{7})$

f) $(\sqrt{6}-3)^2$

5) Work out the following, giving your answer in its simplest form:

a) $\frac{(5+\sqrt{3})(5-\sqrt{3})}{\sqrt{22}}$

b) $\frac{(4-\sqrt{5})(4+\sqrt{5})}{\sqrt{11}}$

c) $\frac{(3-\sqrt{2})(3+\sqrt{2})}{\sqrt{14}}$

d) $\frac{(\sqrt{3}+1)^2}{\sqrt{3}}$

e) $\frac{(\sqrt{5}+3)^2}{\sqrt{20}}$

f) $\frac{(5-\sqrt{5})(2+2\sqrt{5})}{\sqrt{20}}$

Surds

1) $\sqrt{5} = 5^k$

a) Write down the value of k.

b) Expand and simplify $(2 + \sqrt{5})(1 + \sqrt{5})$

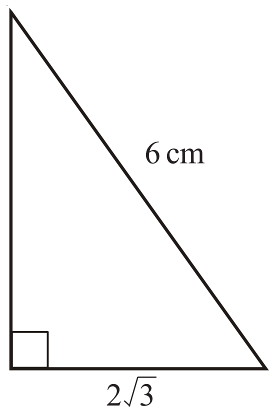
Give your answer in the form $a + b\sqrt{c}$

where a, b and c are integers.

2) The diagram shows a right-angled triangle with lengths of sides as indicated.

The area of the triangle is A cm^2

Show that $A = k\sqrt{2}$ giving the value of k.



3) a) Find the value of $64^{-\frac{2}{3}}$

b) Given that

$$\frac{8 - \sqrt{18}}{\sqrt{2}} = a + b\sqrt{2}\text{, where } a \text{ and } b \text{ are integers,}$$

find the value of a and the value of b.

4) Work out $(2 + \sqrt{3})(2 - \sqrt{3})$

Give your answer in its simplest form.

Rationalising the Denominator

1) Rationalise the denominator, simplifying where possible:

a) $\dfrac{3}{\sqrt{2}}$

b) $\dfrac{2}{\sqrt{2}}$

c) $\dfrac{3\sqrt{2}}{\sqrt{7}}$

d) $\dfrac{\sqrt{5}}{\sqrt{10}}$

e) $\dfrac{1}{4\sqrt{8}}$

f) $\dfrac{\sqrt{15}}{\sqrt{3}}$

g) $\dfrac{1}{\sqrt{27}}$

2) Rationalise the denominator of $\dfrac{1}{\sqrt{3}}$

3) Rationalise the denominator of $\dfrac{1}{8\sqrt{8}}$ giving the answer in the form $\dfrac{\sqrt{2}}{p}$

Direct and Inverse Proportion

1) M is directly proportional to L^3.

When $L = 2$, $M = 160$

Find the value of M when $L = 3$

2) y is directly proportional to x.

When $x = 500$, $y = 10$

a) Find a formula for y in terms of x.

b) Calculate the value of y when $x = 350$

3) D is proportional to S^2.

$D = 900$ when $S = 20$

Calculate the value of D when $S = 25$

4) P is inversely proportional to V.

When $V = 8$, $P = 6$

a) Find a formula for P in terms of V.

b) Calculate the value of P when $V = 2$

5) The time, T seconds, for a hot sphere to cool is proportional to the square root of the surface area, A m^2, of the sphere.

When $A = 100$, $T = 30$.

Find the value of T when $A = 60$.
Give your answer correct to 3 significant figures.

Direct and Inverse Proportion


1) x is directly proportional to y.
When $x = 21$, then $y = 3$.

a) Express x in terms of y.

b) Find the value of x when y is equal to 10.


2) a is inversely proportional to b.
When $a = 12$, then $b = 4$.

a) Find a formula for a in terms of b.

b) Find the value of a when b is equal to 8.

c) Find the value of b when a is equal to 4.


3) The variables u and v are in inverse proportion to one another.
When $u = 3$, then $v = 8$.

Find the value of u when $v = 12$.


4) p is directly proportional to the square of q.
$p = 75$ when $q = 5$

a) Express p in terms of q.

b) Work out the value of p when $q = 7$.

c) Work out the positive value of q when $p = 27$.


5) y is directly proportional to x^2.
When $x = 3$, then $y = 36$.

a) Express y in terms of x.

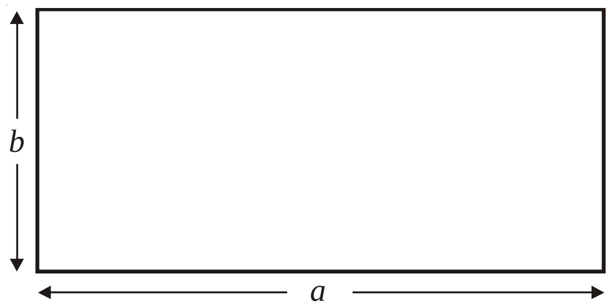
z is inversely proportional to x.
When $x = 4$, $z = 2$.

b) Show that $z = c\,y^n$, where c and n are numbers and $c > 0$.
You must find the values of c and n.

Upper and Lower Bounds

1) Here is a rectangle.

a = 8.4 cm correct to 1 decimal place.
b = 3.6 cm correct to 1 decimal place.



a) Calculate the upper bound of the area of the rectangle.
Write down all the figures on your calculator.

b) Find the area of this rectangle correct to an appropriate number of significant figures.

2) Terry measured the length and the width of a rectangle.

He measured the length to be 745 mm correct to the nearest 5 mm.
He measured the width to be 300 mm correct to the nearest 5 mm.

a) Calculate the lower bound for the area of this rectangle.
Give your answer correct to 3 significant figures.

b) Calculate the upper bound for the perimeter of the rectangle.

3) The voltage V of an electronic circuit is given by the formula

$$\boxed{V = IR}$$

where I is the current in amps
and R is the resistance in ohms.

Given that $V = 217$ correct to three significant figures,
$R = 12.4$ correct to three significant figures,

calculate the lower bound of I.

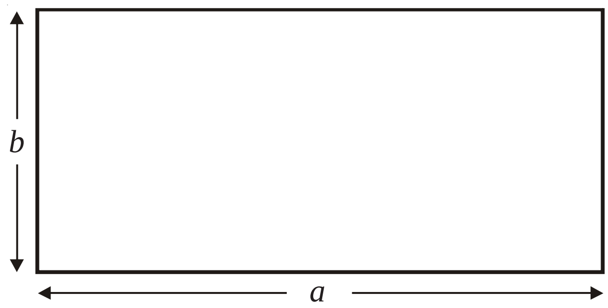
4) Sara drove for 237 miles, correct to the nearest mile.
She used 27.2 litres of petrol, to the nearest tenth of a litre.

$$\boxed{\text{Petrol consumption} = \frac{\text{Number of miles travelled}}{\text{Number of litres of petrol used}}}$$

Work out the upper bound for the petrol consumption for Sara's journey.
Give your answer correct to 2 decimal places.

Upper and Lower Bounds

1)



The length of the rectangle, a,
is 45 cm correct to the nearest cm.

The width of the rectangle, b,
is 26 cm correct to the nearest cm.

Calculate the upper bound for the area of the rectangle.
Write down all the figures on your calculator display.

2) A field is in the shape of a rectangle.
The width of the field is 26 metres, measured to the nearest metre.

a) Work out the upper bound of the width of the field.

The length of the field is 135 metres, measured to the nearest 5 metres.

b) Work out the upper bound for the perimeter of the field.

3) A ball is thrown vertically upwards with a speed V metres per second.

The height, H metres, to which it rises is given by

$$H = \frac{V^2}{2g}$$

where g m/s^2 is the acceleration due to gravity.

$V = 24.4$ correct to 3 significant figures.
$g = 9.8$ correct to 2 significant figures.

(i) Write down the lower bound of g.

(ii) Calculate the upper bound of H.
Give your answer correct to 3 significant figures.

4) $v = \sqrt{\dfrac{a}{b}}$

$a = 6.43$ correct to 2 decimal places.
$b = 5.514$ correct to 3 decimal places.

By considering bounds, work out the value of v to a suitable degree of accuracy.

You must show all your working and give a reason for your final answer.

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Grade A and A questions*	

Upper and Lower Bounds

1) $A = 11.3$ correct to 1 decimal place
$B = 300$ correct to 1 significant figure
$C = 9$ correct to the nearest integer

a) Calculate the upper bound for $A + B$.

b) Calculate the lower bound for $B \div C$.

c) Calculate the least possible value of AC.

d) Calculate the greatest possible value of $\dfrac{A+B}{B+C}$

2) An estimate of the acceleration due to gravity can be found using the formula:

$$g = \frac{2L}{T^2 \sin x}$$

Using
$T = 1.2$ correct to 1 decimal place
$L = 4.50$ correct to 2 decimal places
$x = 40$ correct to the nearest integer

a) Calculate the lower bound for the value of g.
Give your answer correct to 3 decimal places.

b) Calculate the upper bound for the value of g.
Give your answer correct to 3 decimal places.

3) The diagram shows a triangle ABC.

$AB = 73$mm correct to 2 significant figures.
$BC = 80$mm correct to 1 significant figure.

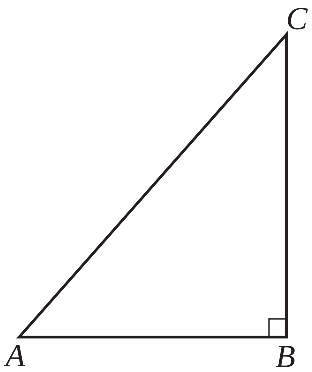


Diagram **NOT** accurately drawn

a) Write the upper and lower bounds of both AB and BC.

AB_{upper} = ... BC_{upper} = ...
AB_{lower} = ... BC_{lower} = ...

b) Calculate the upper bound for the area of the triangle ABC.

........................mm^2

Angle $CAB = x°$

c) Calculate the lower bound for the value of $\tan x°$.

Solve Quadratics Using the Formula

1) Solve the equation $x^2 + 4x + 1 = 0$
Give your answers correct to 3 decimal places.

2) Solve the equation $x^2 + 8x + 6 = 0$
Give your answers correct to 3 significant figures.

3) Solve the equation $x^2 - 3x - 2 = 0$
Give your answers correct to 3 significant figures.

4) Solve the equation $x^2 - 7x + 2 = 0$
Give your answers correct to 3 significant figures.

5) Solve the equation $2x^2 + 6x - 1 = 0$
Give your answers correct to 3 significant figures.

6) Solve the equation $3x^2 - 2x - 20 = 0$
Give your answers correct to 3 significant figures.

7) Solve the equation $x^2 - 14x - 161.25 = 0$

8) Solve the equation $17x^2 - 92x - 206 = 0$
Give your answers correct to 3 significant figures.

9) $x^2 + 10x = 300$
Find the positive value of x.
Give your answer correct to 3 significant figures.

10) $(x + 2)(x - 3) = 1$

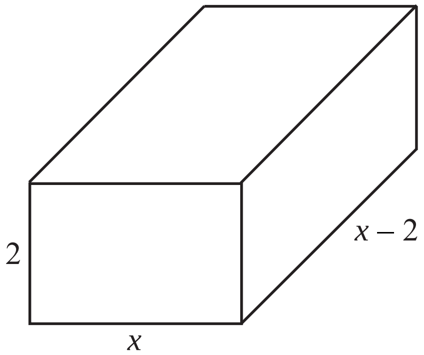
a) Show that $x^2 - x - 7 = 0$

b) Solve the equation $x^2 - x - 7 = 0$

Give your answers correct to 3 significant figures.

Solve Quadratics Using the Formula

1)



The diagram shows a cuboid.
All the measurements are in cm.

The volume of the cuboid is 52 cm^3.

a) Show that $2x^2 - 4x - 52 = 0$ for $x > 2$

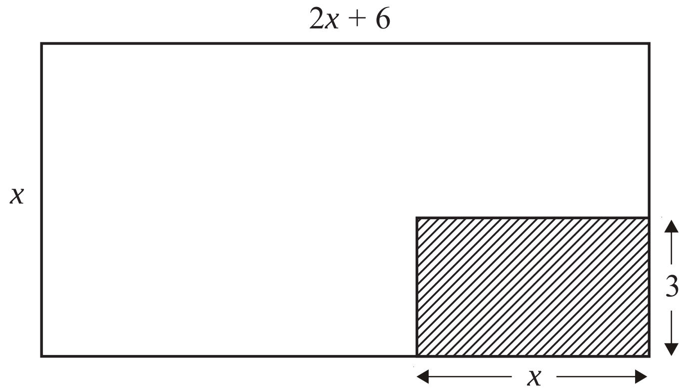
b) Solve the quadratic equation

$$2x^2 - 4x - 52 = 0$$

Give your solutions correct to 3 significant figures.
You must show your working.

2) The diagram below shows a large rectangle of length $(2x + 6)$ cm and width x cm.

A smaller rectangle of length x cm and width 3 cm is cut out and removed.



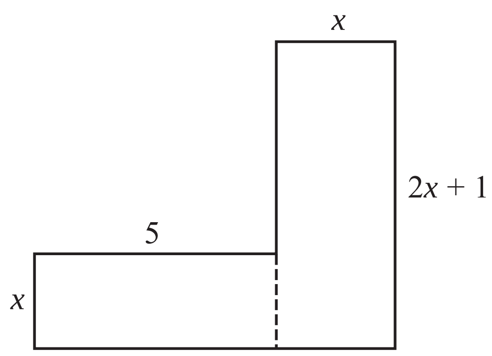
The area of the shape that is left is 100 cm^2.

a) Show that $2x^2 + 3x - 100 = 0$

b) Calculate the length of the smaller rectangle.
Give your answer correct to 3 significant figures.

Solve Quadratics Using the Formula

1)



The diagram shows a 6-sided shape.
All the corners are right angles.
All the measurements are given in centimetres.

The area of the shape is 94 cm².

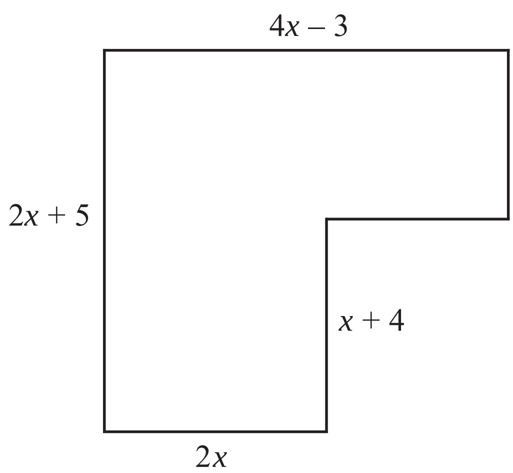
a) Show that $2x^2 + 6x - 94 = 0$

b) Solve the equation

$$2x^2 + 6x - 94 = 0$$

Give your solutions correct to 3 significant figures.

2) The diagram shows a 6-sided shape.
All the corners are right angles.



All the measurements are given in centimetres.

The area of the shape is 33 cm².

Work out the length of the longest side of the shape.
Give your answer correct to 2 significant figures.

Completing the Square

1) Show that if $y = x^2 + 8x - 3$
then $y \geq -19$ for all values of x.

2) Show that if $y = x^2 - 10x + 30$
then $y \geq 5$ for all values of x.

3) The expression $x^2 + 4x + 10$ can be written in the form
$(x + p)^2 + q$ for all values of x.
Find the values of p and q.

4) Given that $x^2 - 6x + 17 = (x - p)^2 + q$ for all values of x,
find the value of p and the value of q.

5) For all values of x,

$$x^2 + 6x = (x + p)^2 + q$$

a) Find the values of p and q.

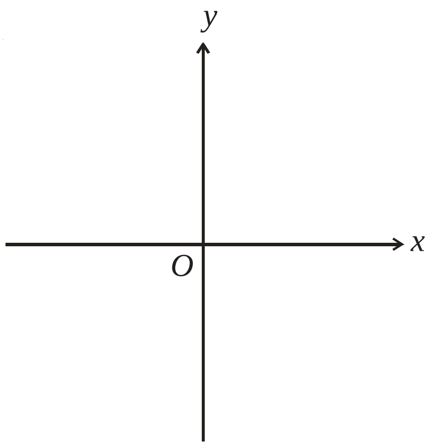
b) Find the minimum value of $x^2 + 6x$.

6) For all values of x,

$$x^2 - 8x - 5 = (x - p)^2 + q$$

a) Find the value of p and the value of q.

b) On the axes, sketch the graph of $y = x^2 - 8x - 5$.



c) Find the coordinates of the minimum point on the graph of $y = x^2 - 8x - 5$.

7) The expression $10x - x^2$ can be written in the form $p - (x - q)^2$ for all values of x.

a) Find the values of p and q.

b) The expression $10x - x^2$ has a maximum value.

(i) Find the maximum value of $10x - x^2$.

(ii) State the value of x for which this maximum value occurs.

Algebraic Fractions

1) Simplify fully

a) $\dfrac{9x^2}{21x^3}$

b) $\dfrac{10xy^3}{5y^2}$

c) $\dfrac{18a^3b^2}{2ab^2}$

d) $\dfrac{4x^2+12x}{10x}$

e) $\dfrac{2a^2b-14a^2b^3}{6a^3b^3}$

f) $\dfrac{5x^2y+5xy^2}{10x^2y^2}$

2) Simplify fully

a) $\dfrac{x^2+x}{x^2+6x+5}$

b) $\dfrac{x^2-6x+8}{2x^2-8x}$

c) $\dfrac{x^2+7x+10}{x^2+5x}$

3) a) Factorise $4x^2-12x+9$

b) Simplify $\dfrac{6x^2-7x-3}{4x^2-12x+9}$

Algebraic Fractions

1) Write as single fractions in their simplest form

a) $\dfrac{3}{x}+\dfrac{3}{2x}$

b) $\dfrac{5}{3x}-\dfrac{3}{4x}$

c) $\dfrac{x+2}{5}+\dfrac{x-1}{2}$

d) $\dfrac{3}{x+2}-\dfrac{5}{2x+1}$

2) a) Factorise $2x^2+7x+6$

b) Write as a single fraction in its simplest form $\dfrac{3}{x+2}+\dfrac{4x}{2x^2+7x+6}$

3) Solve

a) $\dfrac{1}{x}+\dfrac{1}{3x}=2$

b) $\dfrac{1}{x-2}+\dfrac{3}{x+6}=\dfrac{1}{2}$

c) $\dfrac{1}{x-5}+\dfrac{6}{x}=2$

d) $\dfrac{7}{x+2}+\dfrac{1}{x-1}=4$

e) $\dfrac{3}{x+2}+\dfrac{1}{x-2}=\dfrac{7}{x^2-4}$

f) $\dfrac{x}{2x-1}+\dfrac{2}{x+2}=1$

Rearranging Difficult Fomulae

1) Make c the subject of the formula.

$v = 2a + 3b + c$

2) Make t the subject of the formula.

$A = \pi t + 5t$

3) Make s the subject of the formula.

$R = 3s + \pi s + 2t$

4) $k = \dfrac{l}{m - l}$

a) Make l the subject of the formula.

b) Make m the subject of the formula.

5) $A = \dfrac{k(x+5)}{3}$

Make x the subject of the formula.

6) $R = \dfrac{u + v^2}{u + v}$

Make u the subject of the formula.

7) $\dfrac{3x+2}{5} = \dfrac{y}{10+y}$

Make y the subject of the formula.

8) $\sqrt{\dfrac{a-3}{5}} = 4b$

Rearrange this formula to give a in terms of b.

9) $S = 2\pi d\sqrt{h^2 + d^2}$

Rearrange this formula to make h the subject.

Simultaneous Equations with a Quadratic

1) Solve these simultaneous equations.

$y = x$

$y = x^2 - 6$

2) Solve these simultaneous equations.

$y = x^2 - 4$

$y = 3x$

3) Solve these simultaneous equations.

$y = x^2 - x - 13$

$y = x + 2$

4) Solve these simultaneous equations.

$y = x^2 - 35$

$x - y = 5$

5) Solve these simultaneous equations.

$x^2 + y^2 = 26$

$y + 6 = x$

6) Sarah said that the line $y = 7$ cuts the curve $x^2 + y^2 = 25$ at two points.

a) By eliminating y show that Sarah is **not** correct.

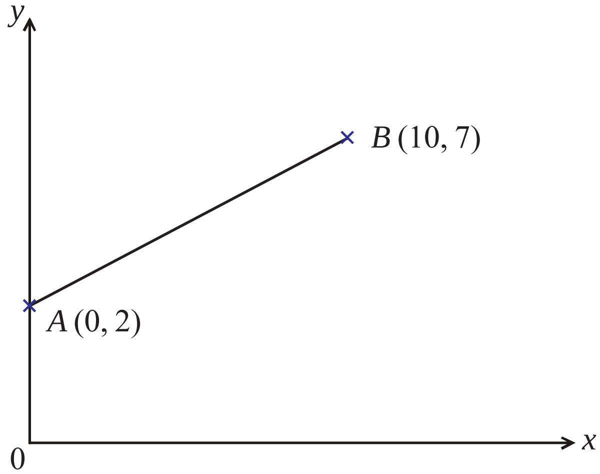
b) By eliminating y, find the solutions to the simultaneous equations

$x^2 + y^2 = 25$

$y = 3x - 9$

Gradients of Lines

1)



A is the point (0, 2)
B is the point (10, 7)

a) Write down the equation of the straight line which passes through points A and B.

b) Find the equation of the line perpendicular to AB passing through B.

2) A straight line has equation $y = 2x - 5$
The point P lies on the straight line.
The y coordinate of P is -6

a) Find the x coordinate of P.

A straight line ***L*** is parallel to $y = 2x - 5$ and passes through the point (3, 2).

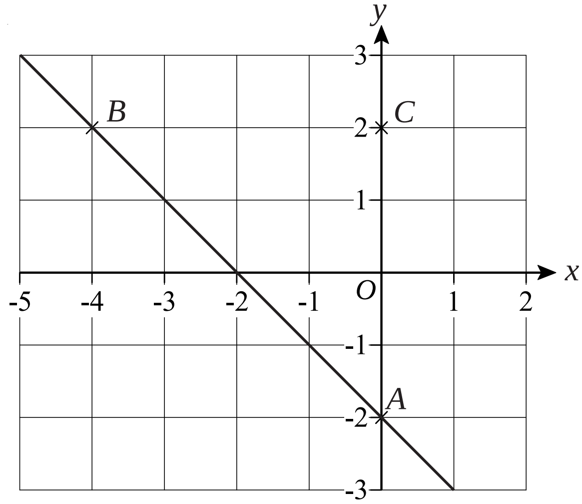
b) Find the equation of line ***L***.

c) Find the equation of the line that is perpendicular to line ***L*** and passes through point (3, 2).

3) In the diagram A is the point (0, -2)
B is the point (-4, 2)
C is the point (0, 2)

a) Find the equation of the line that passes through C and is parallel to AB.

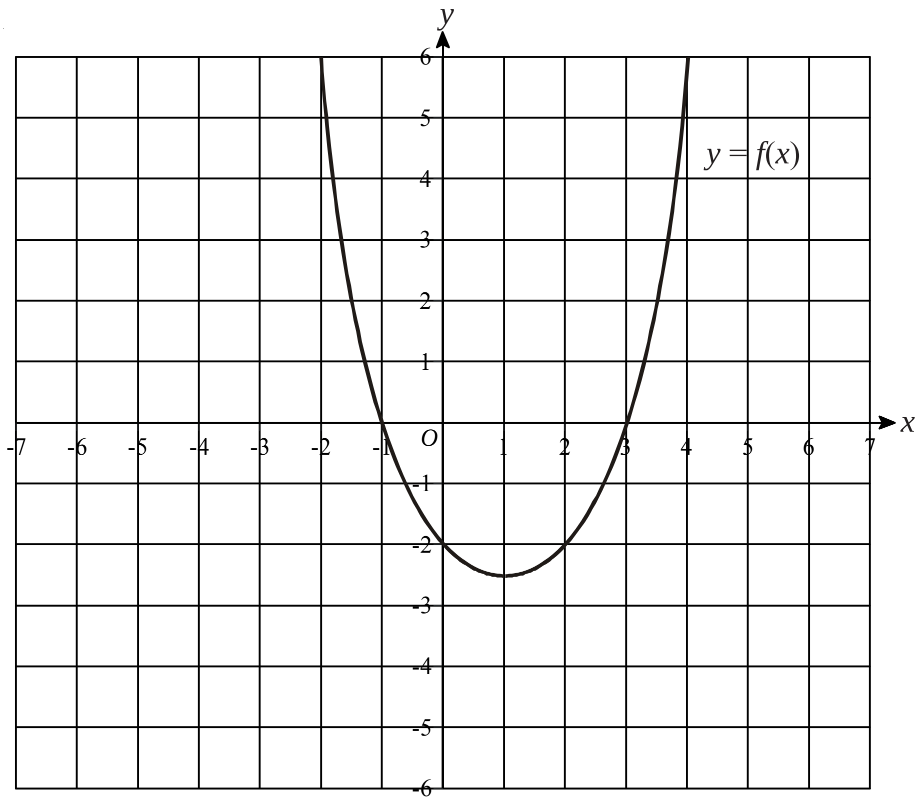
b) Find the equation of the line that passes through C and is perpendicular to AB.



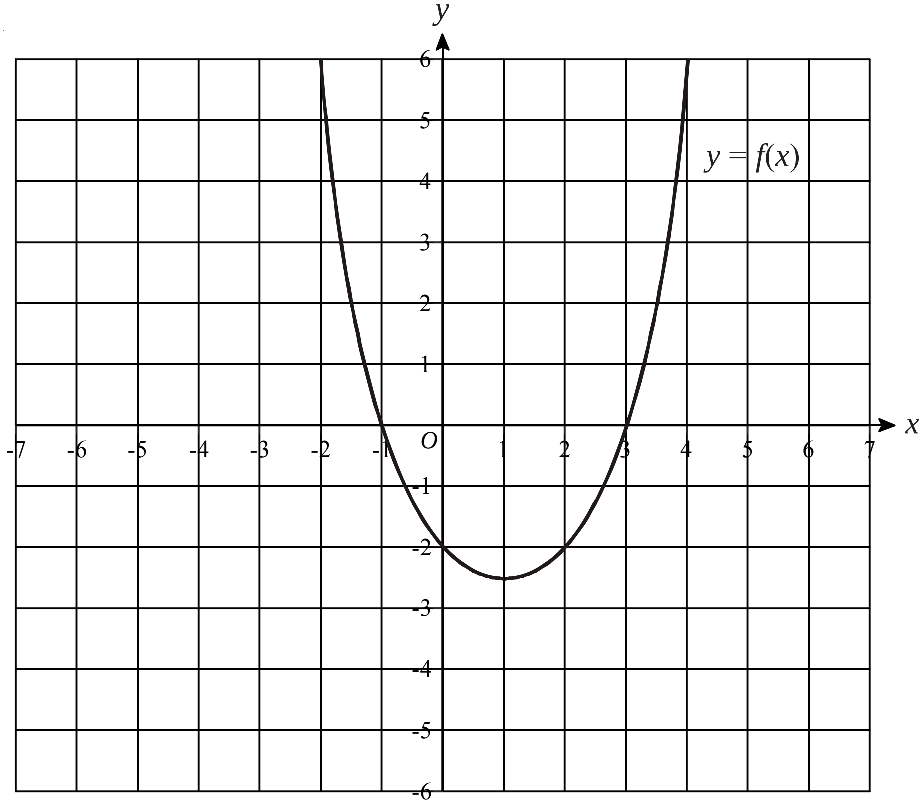
Transformations of Functions

1) The graph of $y = f(x)$ is shown on the grids.

a) On this grid, sketch the graph of $y = f(x - 3)$

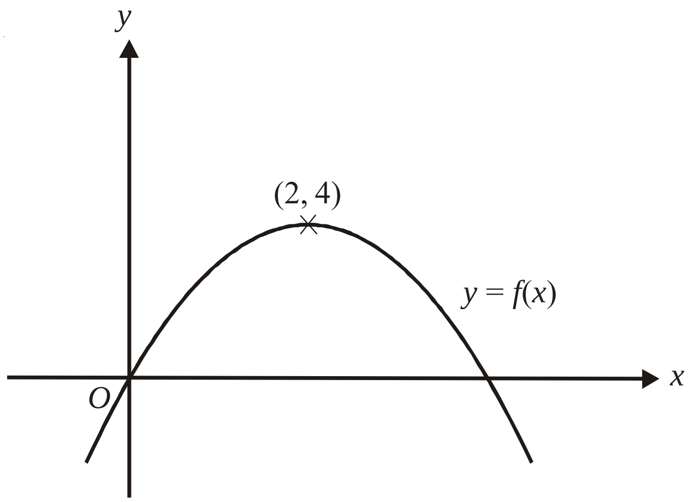


b) On this grid sketch the graph of $y = -f(x)$



Transformations of Functions

1)



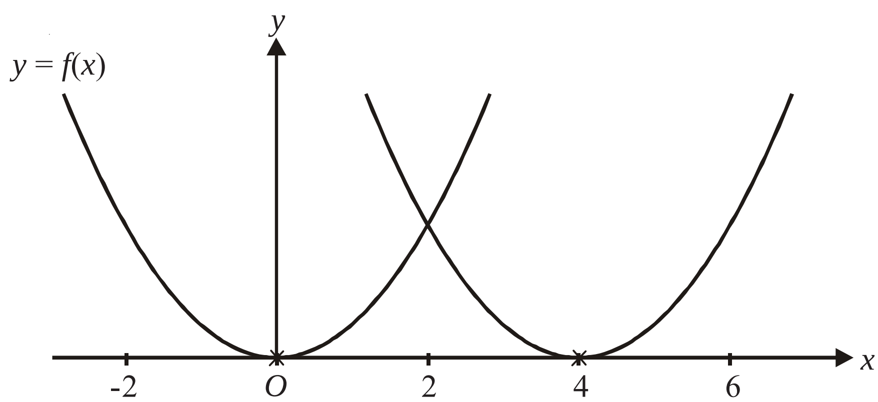
The diagram shows part of the curve with equation $y = f(x)$.
The coordinates of the maximum point of this curve are (2, 4).

Write down the coordinates of the maximum point of the curve with equation

a) $y = f(x - 2)$

b) $y = 2f(x)$

2)

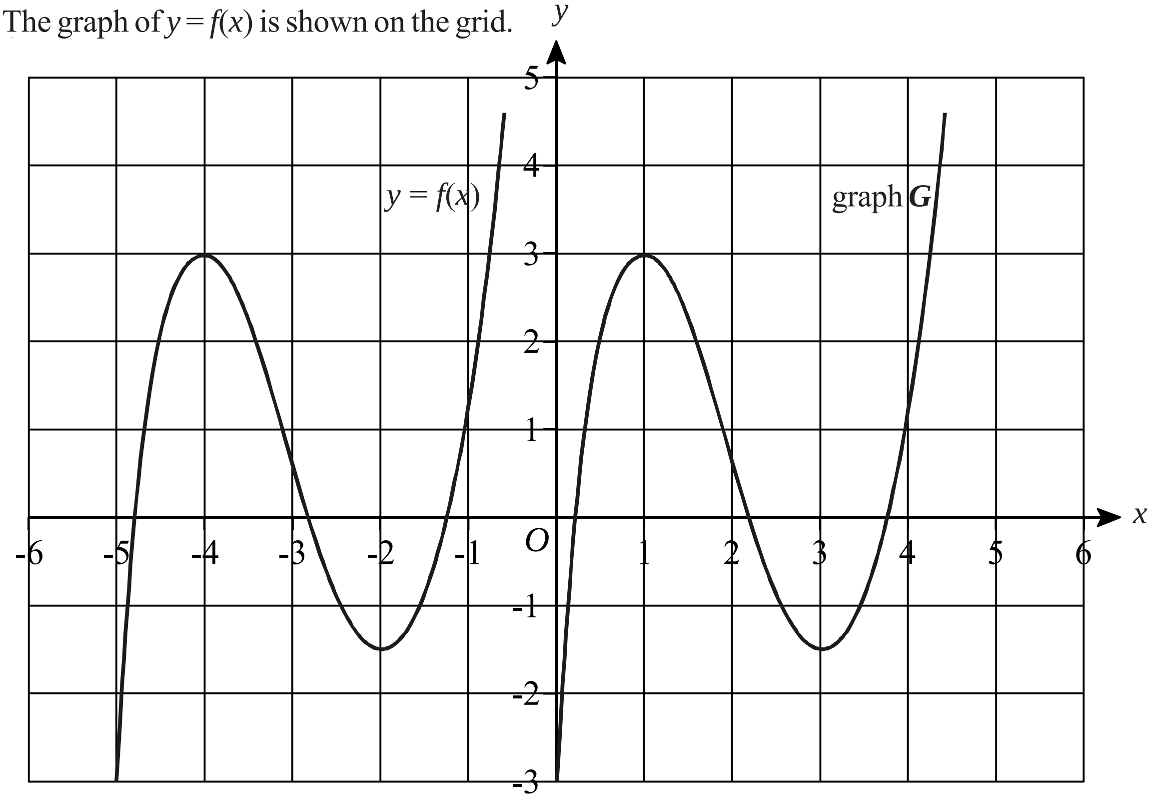


The curve with equation $y = f(x)$ is translated so that the point at (0, 0) is mapped onto the point (4, 0).

Find the equation of the translated curve.

Transformations of Functions

1) The graph of $y = f(x)$ is shown on the grid.



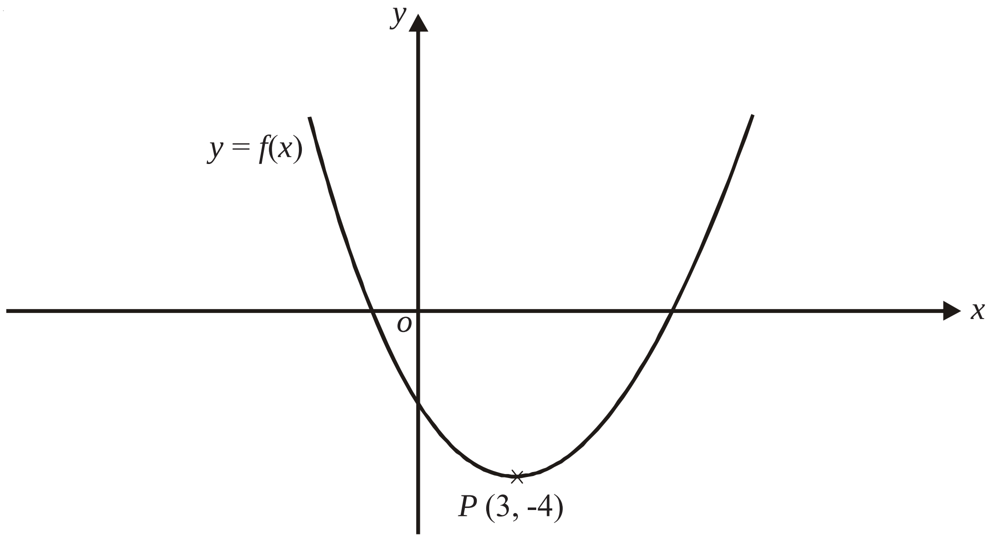
The graph **G** is a translation of the graph of $y = f(x)$.

a) Write down, in terms of f, the equation of graph **G**.

The graph of $y = f(x)$ has a maximum point at (-4, 3).

b) Write down the coordinates of the maximum point of the graph $y = f(-x)$.

2) This is a sketch of the curve with the equation $y = f(x)$.
The only minimum point of the curve is at P (3, -4).

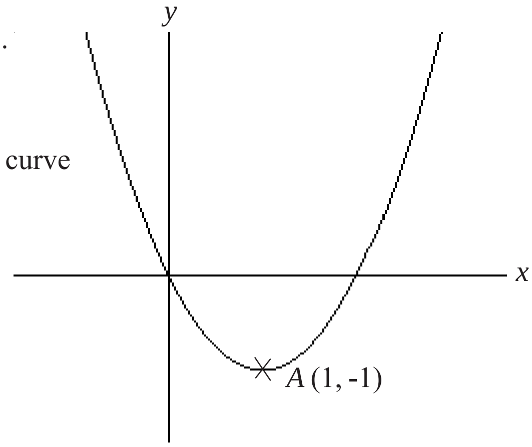


a) Write down the coordinates of the minimum point of the curve with the equation $y = f(x - 2)$

b) Write down the coordinates of the minimum point of the curve with the equation $y = f(x + 5) + 6$

Transformations of Functions

1) This is a sketch of the curve with equation $y = f(x)$.
It passes through the origin O.
The only vertex of the curve is at A (1, -1)



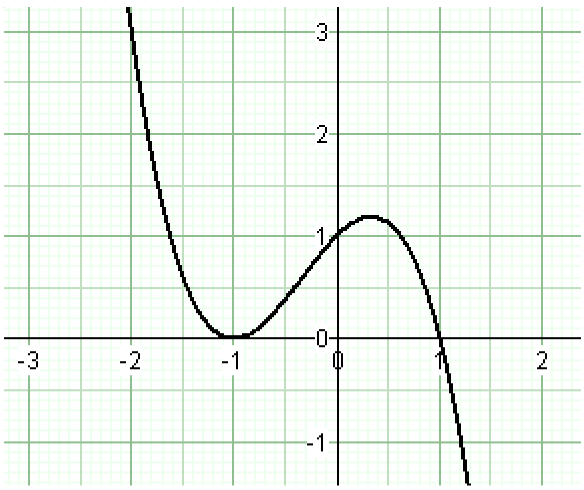
a) Write down the coordinates of the vertex of the curve with equation

(i) $y = f(x - 3)$
(ii) $y = f(x) - 5$
(iii) $y = -f(x)$
(iv) $y = f(2x)$

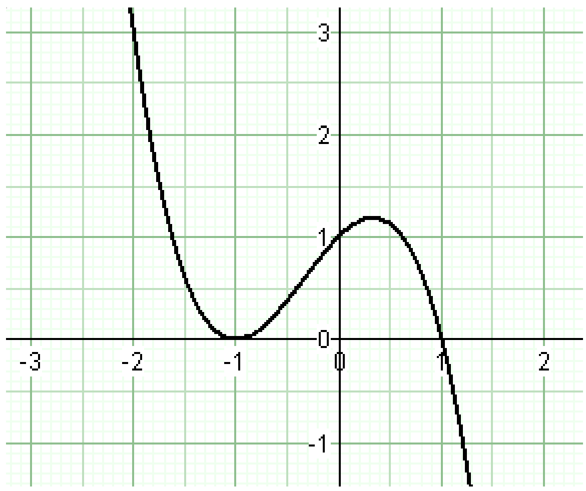
b) The curve $y = x^2$ has been translated to give the curve $y = f(x)$.
Find $f(x)$ in terms of x.

2) The graph of $y = f(x)$ is shown on the grids.

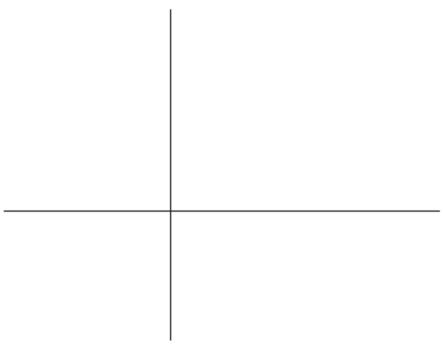
a) On this grid, sketch the graph of $y = f(x - 1)$



b) On this grid, sketch the graph of $y = 2f(x)$

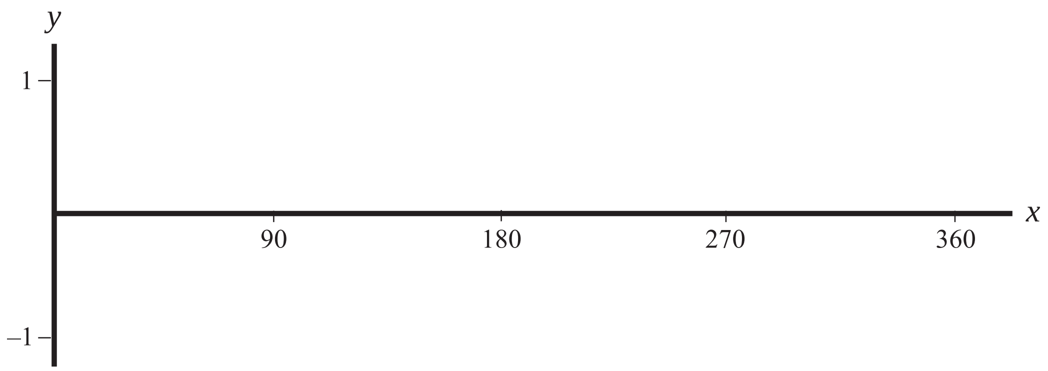


3) Sketch the graph of $y = (x - 2)^2 + 3$
State the coordinates of the vertex.



Graphs of Trigonometric Functions

1) On the axes below below, draw a sketch-graph to show $y = \sin x$

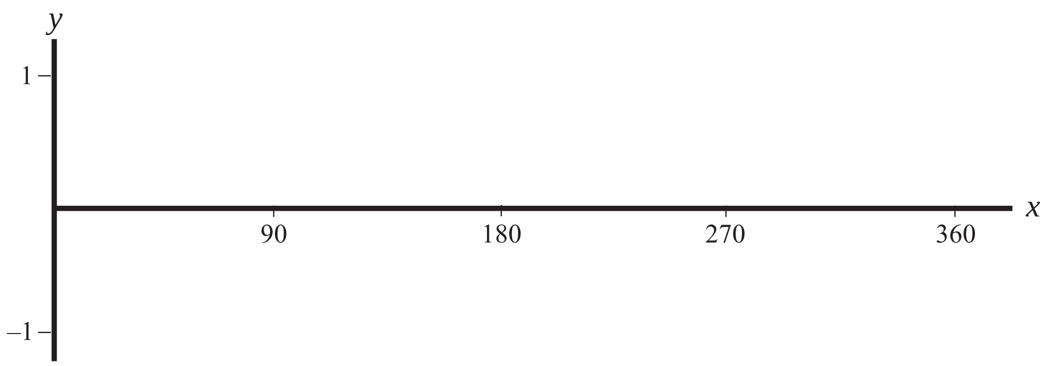


Given that sin 30° = 0.5, write down the value of:

(i) sin 150°

(ii) sin 330°

2) On the axes below, draw a sketch-graph to show $y = \cos x$



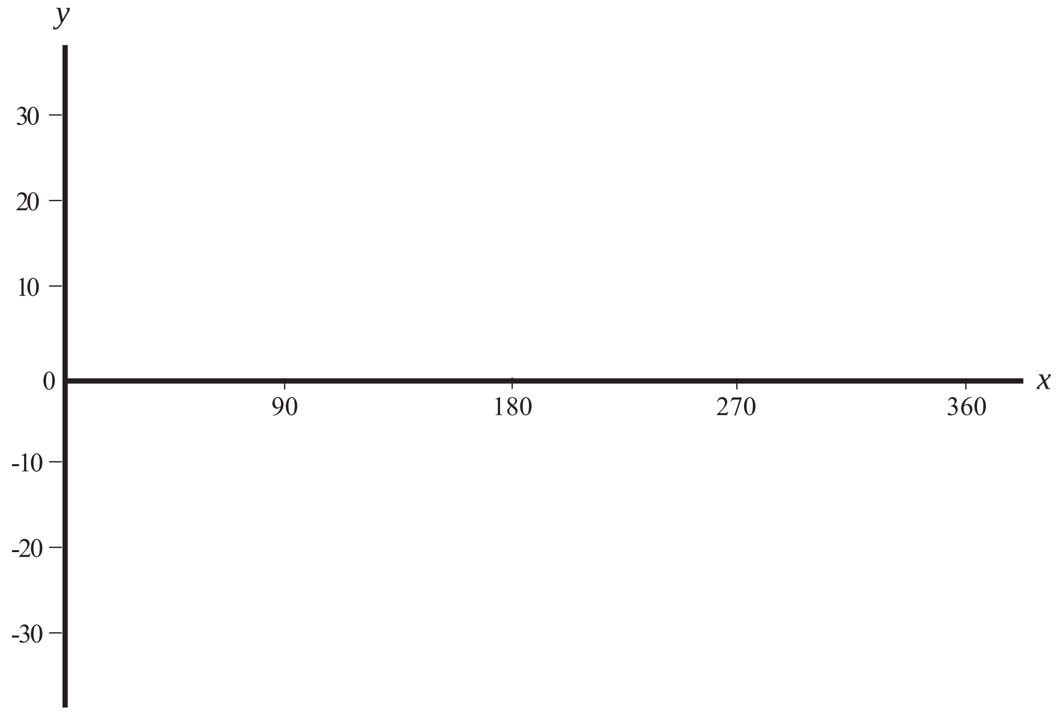
Given that cos 60° = 0.5, write down the value of:

(i) cos 120°

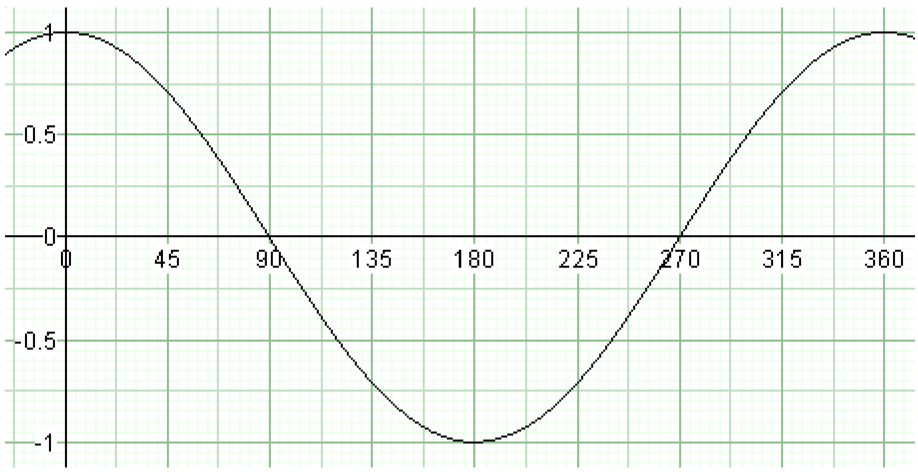
(ii) cos 240°

Graphs of Trigonometric Functions

1) On the axes below, draw a sketch-graph to show $y = \tan x$



2) Here is the graph of the curve $y = \cos x$ for $0 \leqslant x \leqslant 360^\circ$.

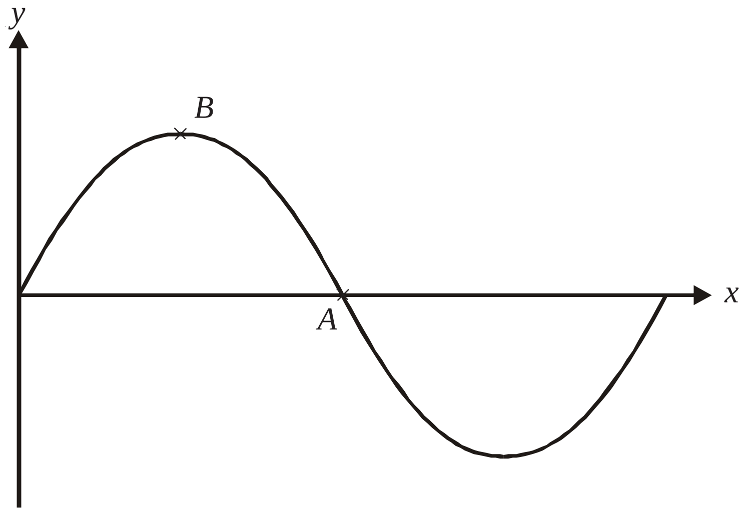


a) Use the graph to solve $\cos x = 0.75$ for $0 \leqslant x \leqslant 360^\circ$

b) Use the graph to solve $\cos x = -0.75$ for $0 \leqslant x \leqslant 360^\circ$

Transformations of Trigonometric Functions

1) The diagram below shows the graph of $y = 2\sin x$, for values of x between 0 and 360°.



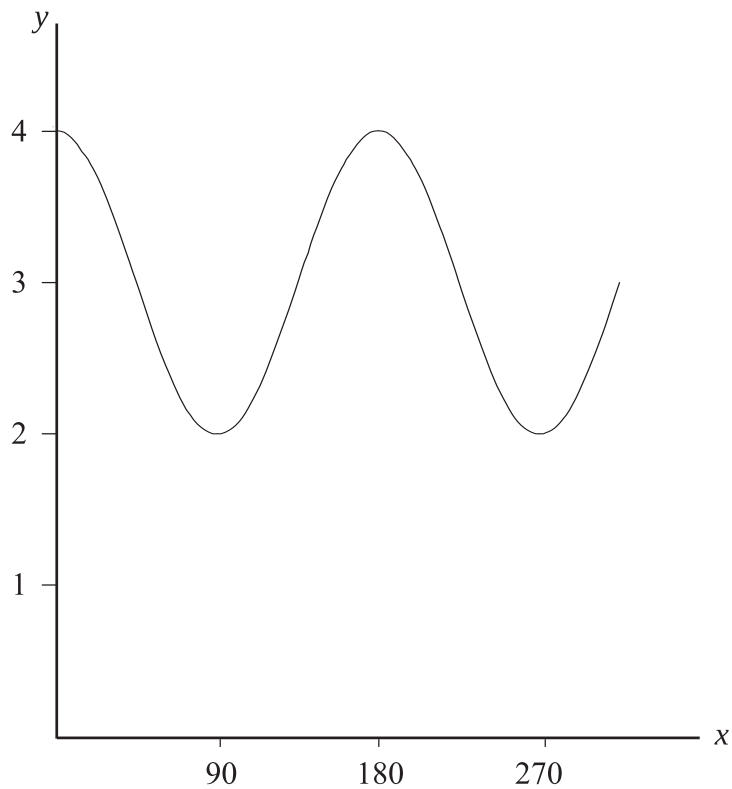
The curve cuts the x axis at the point A.
The graph has a maximum at the point B.

a) (i) Write down the coordinates of A.
(ii) Write down the coordinates of B.

b) On the same diagram, sketch the graph of $y = 2\sin x + 1$ for values of x between 0° and 360°.

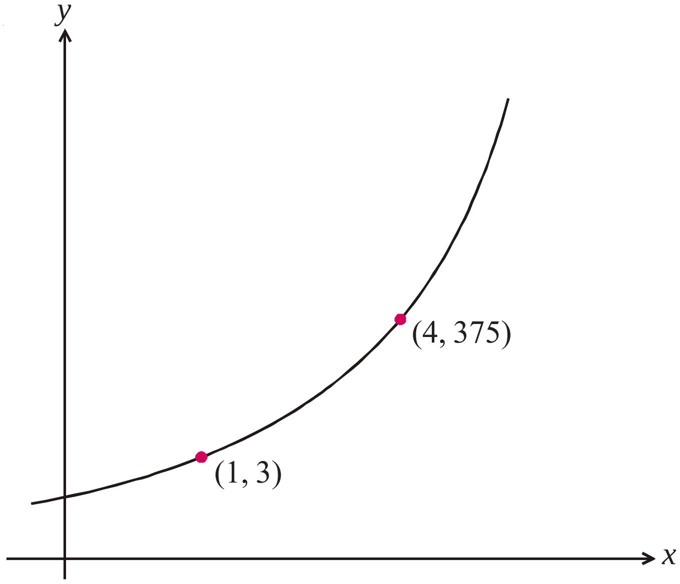
2) The diagram below shows the graph of $y = \cos \boldsymbol{a}x + \boldsymbol{b}$, for values of x between 0° and 300°.

Work out the values of $\boldsymbol{a}$ and $\boldsymbol{b}$.



Graphs of Exponential Functions

1)



The sketch-graph shows a curve with equation $y = pq^x$.

The curve passes through the points (1, 3) and (4, 375).

Calculate the value of p and the value of q.

2) The graph shows the number of bacteria living in a petri dish.
The number N of bacteria at time t is given by the relation:

$$N = a \times b^t$$

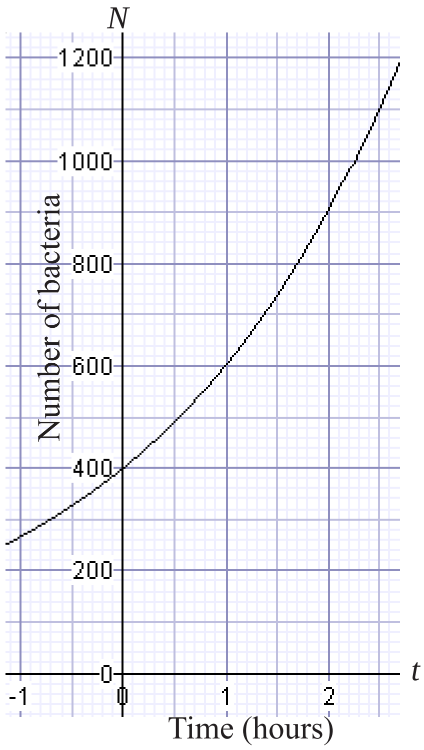
The curve passes through the point (0, 400).

a) Use this information to show that $a = 400$.

The curve also passes through (2, 900).

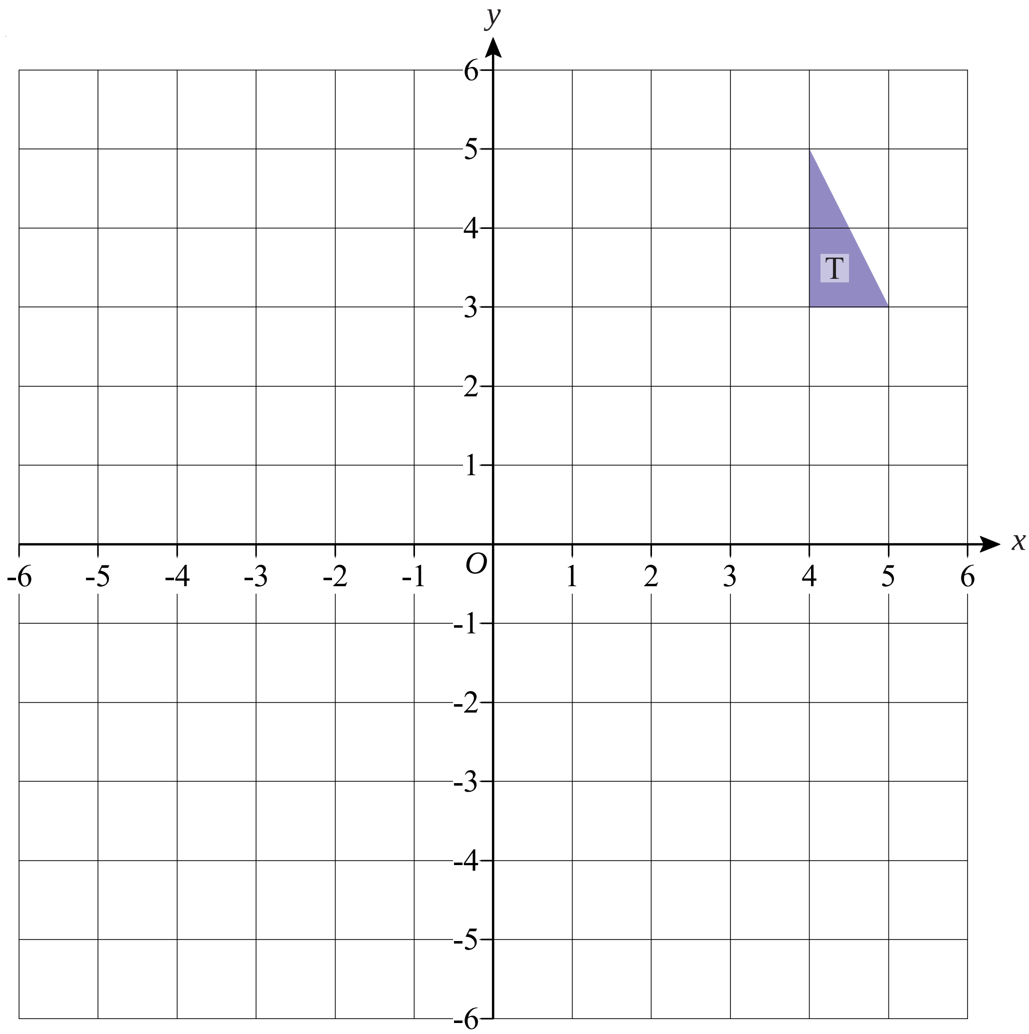
b) Use this information to find the value of b.

c) Work out the number of bacteria in the dish at time $t = 3$.

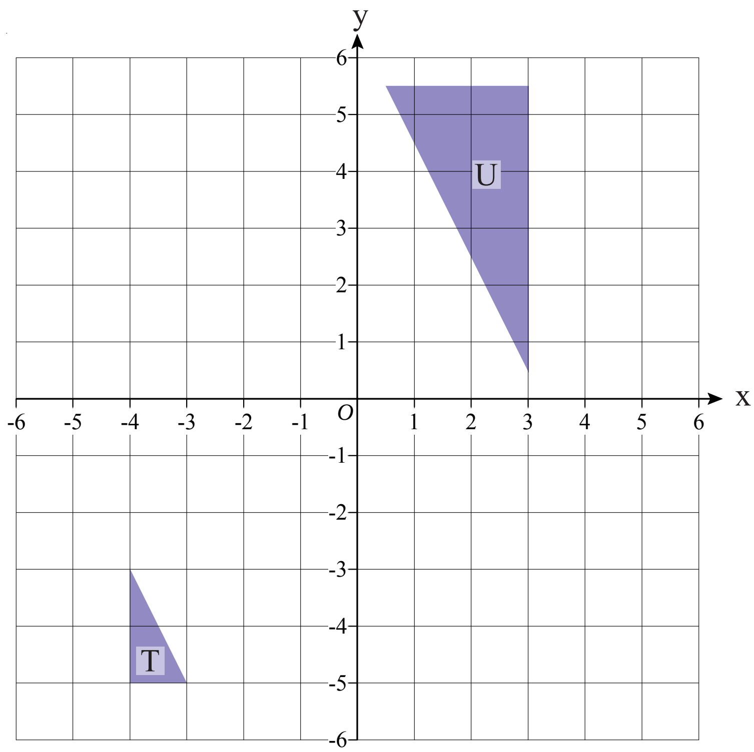


Enlargements by Negative Scale Factors

1) Enlarge triangle T by scale factor -2 using coordinates (2, 2) as the centre of enlargement.

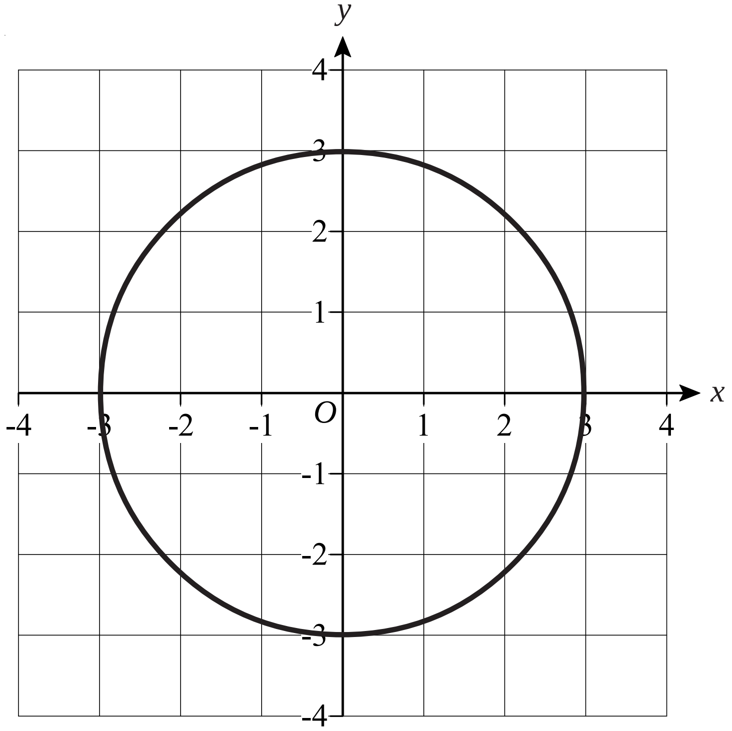


2) Describe fully the single transformation which maps triangle T to triangle U.

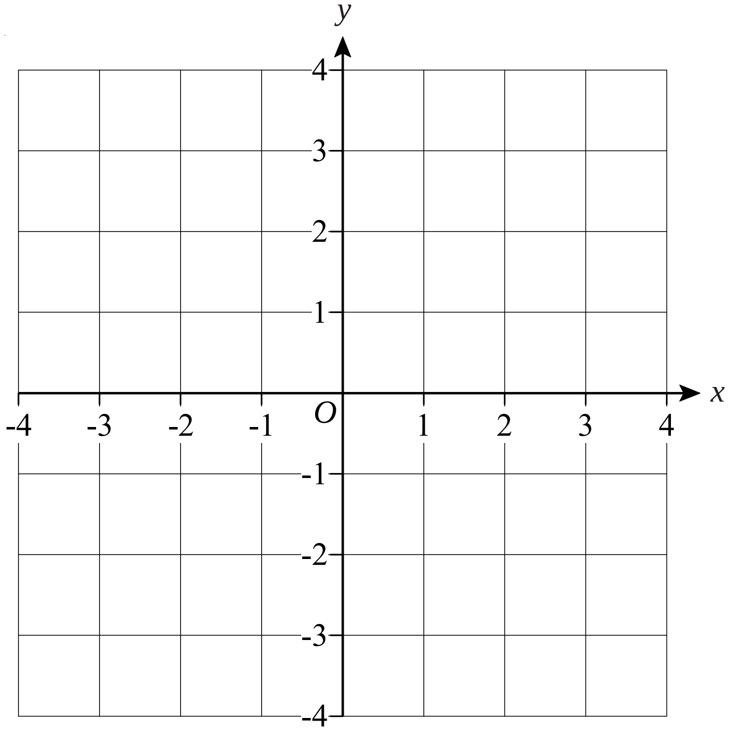


Equations of Circles

1) Find the equation of a circle with radius 3 and centre the origin.



2) a) Draw the graph of $x^2 + y^2 = 6.25$

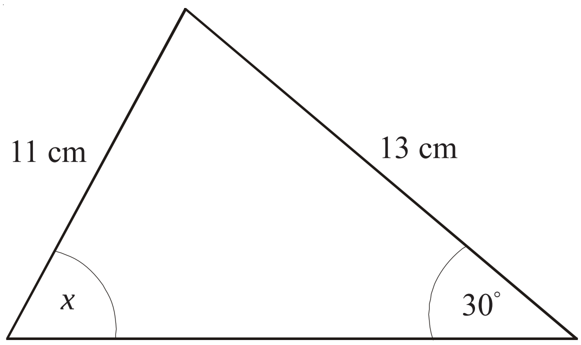


b) By drawing the line $x + y = 1.5$, solve the equations

$$x^2 + y^2 = 6.25$$
$$x + y = 1.5$$

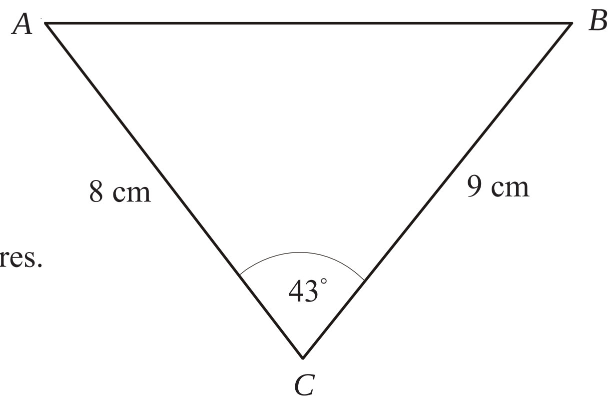
Sine and Cosine Rules

1) Work out the size of the angle marked x.
Give your answer correct to one decimal place.



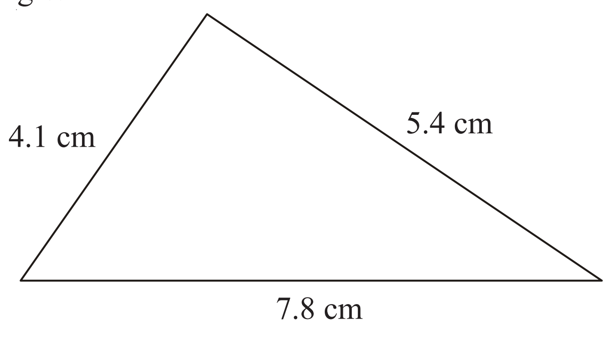
2) ABC is a triangle.
$AC = 8$ cm
$BC = 9$ cm
Angle $ACB = 43^\circ$

Calculate the length of AB.
Give your answer correct to 3 significant figures.



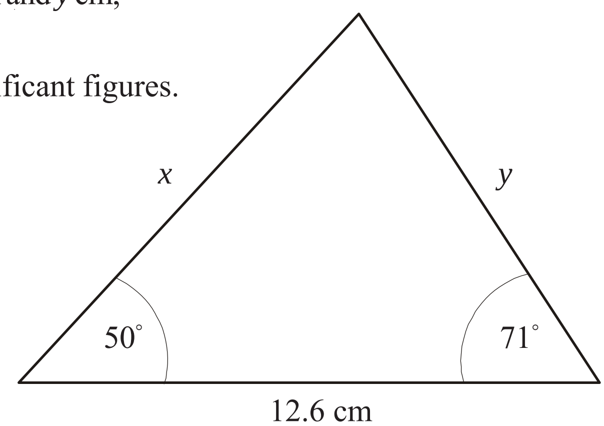
3) The lengths of the sides of a triangle are 4.1 cm, 5.4 cm and 7.8 cm.

Calculate the size of the largest angle of the triangle.
Give your answer correct to 1 decimal place.

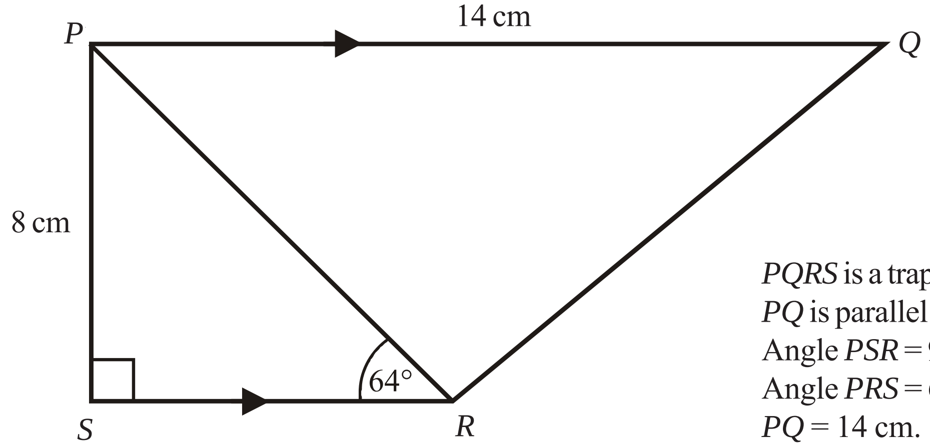


4) Find the missing lengths, x cm and y cm,
in this triangle.

Give your answers to 3 significant figures.



Sine and Cosine Rules

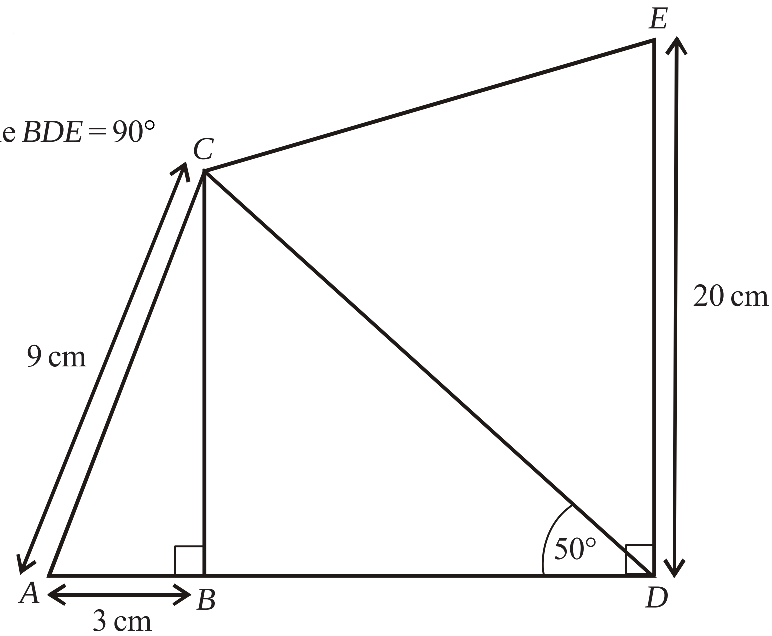
1) 

PQRS is a trapezium.
PQ is parallel to *SR*.
Angle *PSR* = 90°
Angle *PRS* = 64°
PQ = 14 cm.
PS = 8 cm.

a) Work out the length of *PR*.
Give your answer correct to 3 significant figures.

b) Work out the length of *QR*.
Give your answer correct to 3 significant figures.

2) *AC* = 9 cm
AB = 3 cm
DE = 20 cm
Angle *ABC* = angle *CBD* = angle *BDE* = 90°



a) Calculate the length of *CD*.
Give your answer to 3 significant figures.

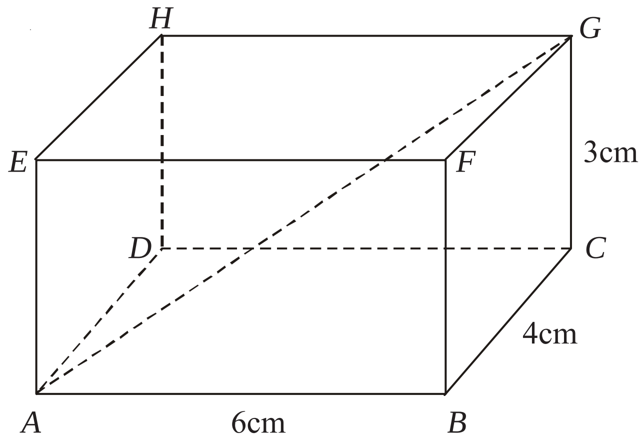
b) Calculate the length of *CE*.
Give your answer to 3 significant figures.

Pythagoras in 3D

1) The diagram shows a box in the shape of a cuboid.
$AB = 6\text{cm}$, $BC = 4\text{cm}$, $CG = 3\text{cm}$

A string runs diagonally across the box from A to G.

Calculate the length of the string AG.
Give your answer correct to 3 significant figures.



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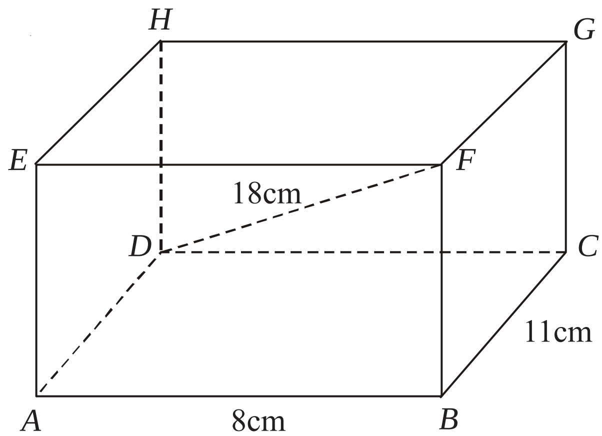
2) The diagram shows a box in the shape of a cuboid.
$AB = 8\text{cm}$, $BC = 11\text{cm}$

A string runs diagonally across the box from D to F and is 18cm long.

Calculate the length AE.
Give your answer correct to 3 significant figures.



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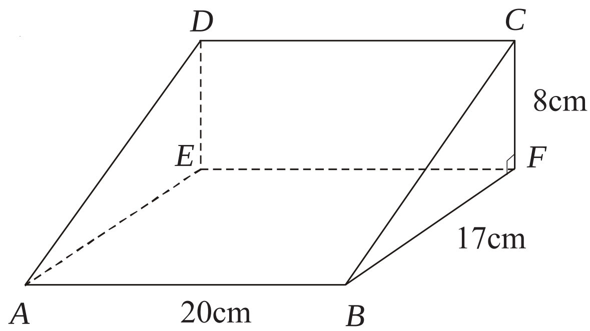
3) The diagram shows a wedge in the shape of a prism.
Angle BFC is a right angle.

String runs diagonally across the wedge from A to C.

Calculate the length AC
Give your answer correct to 3 significant figures.



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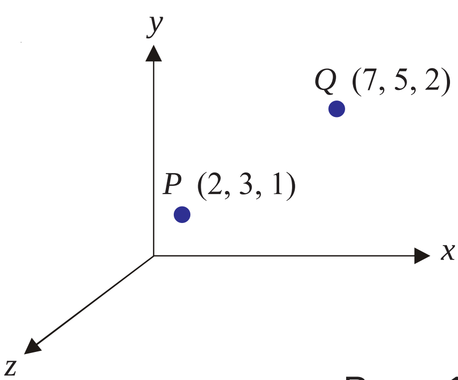
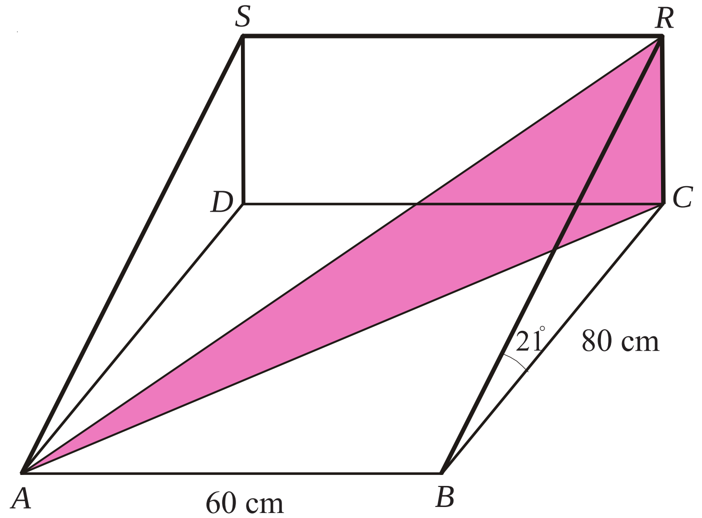
4) Two points, P and Q, lie on coordinate axes.

Find the distance PQ to 1 decimal place.



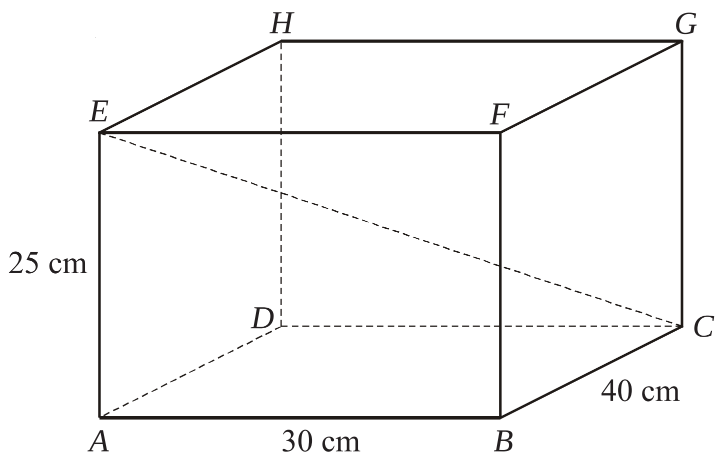
Trigonometry in 3D

1) The diagram shows a wedge.
The base of the wedge is a horizontal rectangle measuring 80 cm by 60 cm.
The sloping face *ABRS* makes an angle of 21° to the horizontal.



Calculate the angle that *AR* makes with the horizontal plane *ABCD*.
Give your answer correct to 1 decimal place.

2) The diagram shows a box in the shape of a cuboid.
A string runs diagonally across the box from *C* to *E*.



a) Work out the length of the string *CE*.
Give your answer correct to 1 decimal place.

b) Work out the angle between the string *CE* and the horizontal plane *ABCD*.
Give your answer correct to 1 decimal place.

Areas of Triangles Using ½*ab*Sin*C*

1)

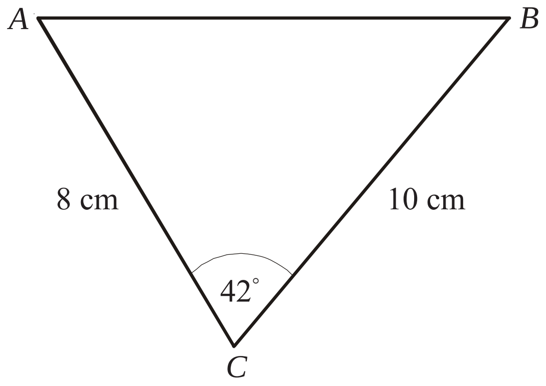


Diagram **NOT** accurately drawn.

ABC is a triangle.
AC = 8 cm.
BC = 10 cm
Angle *ACB* = 42°

Calculate the area of triangle *ABC*.
Give your answer correct to 3 significant figures.

2)

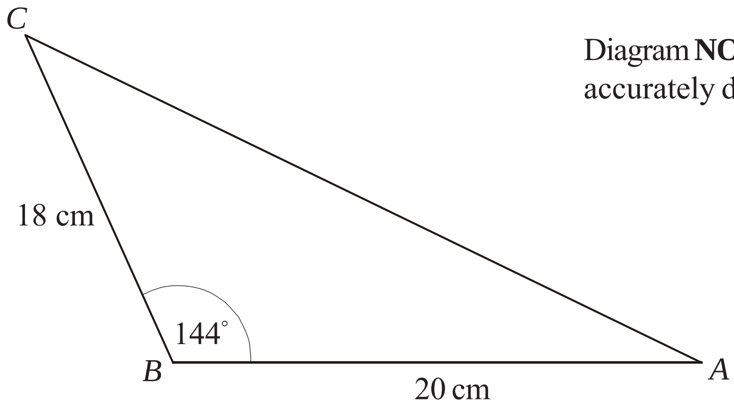


Diagram **NOT** accurately drawn.

ABC is a triangle.
AB = 20 cm.
BC = 18 cm
Angle *ABC* = 144°

Calculate the area of triangle *ABC*.
Give your answer correct to 3 significant figures.

3)

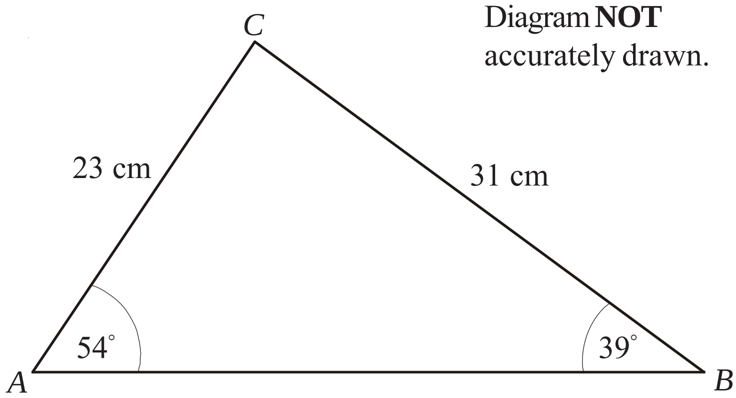


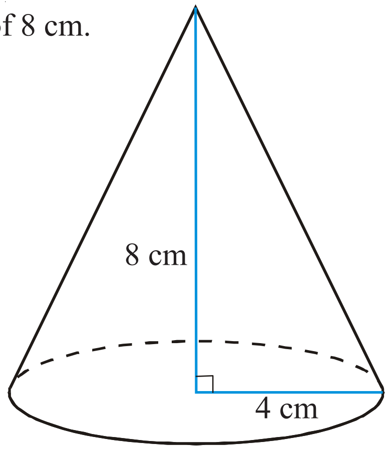
Diagram **NOT** accurately drawn.

ABC is a triangle.
AC = 23 cm.
BC = 31 cm
Angle *BAC* = 54°
Angle *ABC* = 39°

Calculate the area of triangle *ABC*.
Give your answer correct to 3 significant figures.

Cones and Spheres

1) A cone has a base radius of 4 cm and a vertical height of 8 cm.

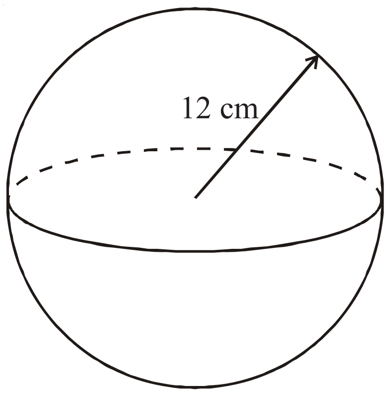


a) Calculate the volume of the cone.
Take π to be 3.142.
Give your answer correct to 3 significant figures.

b) Use Pythagoras' Theorem to find the slant height of the cone.
Give your answer correct to 1 decimal place.

c) Find the curved surface area of the cone.
Take π to be 3.142.
Give your answer correct to 3 significant figures.

2) A sphere has a radius of 12cm.

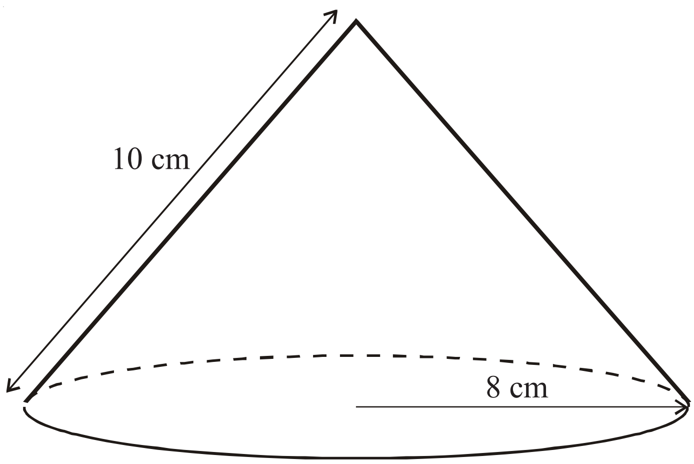


a) Calculate the volume of the sphere.
Take π to be 3.142.
Give your answer correct to 3 significant figures.

b) Find the curved surface area of the sphere.
Take π to be 3.142.
Give your answer correct to 3 significant figures.

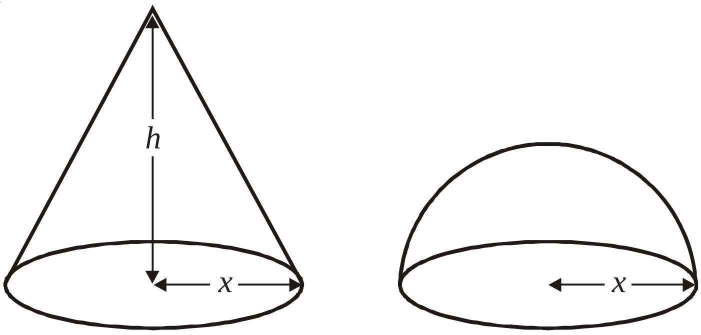
3) A cone has a base radius of 8 cm and a slant height of 10 cm.

Calculate the volume of the cone.
Leave your answer in terms of π.



Cones and Spheres

1)

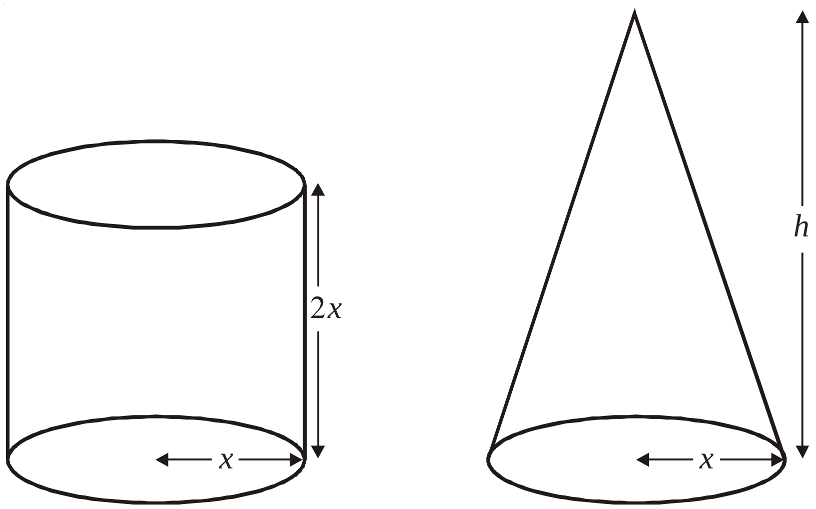


The diagram shows a solid cone and a solid hemisphere.

The cone has a base of radius x cm and a height of h cm.
The hemisphere has a base of radius x cm.
The surface area of the cone is equal to the surface area of the hemisphere.

Find an expression for h in terms of x.

2)



A cylinder has base radius x cm and height $2x$ cm.

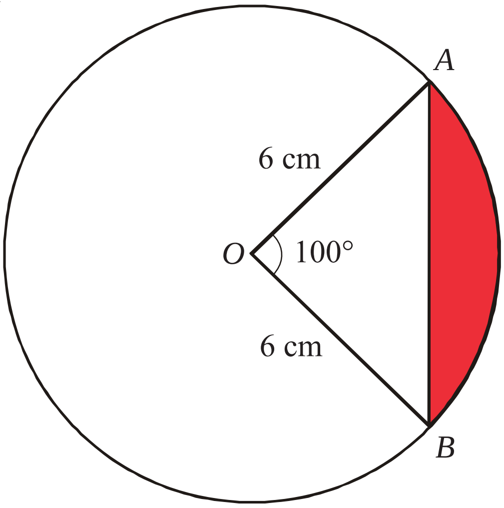
A cone has base radius x cm and height h cm.

The volume of the cylinder and the volume of the cone are equal.

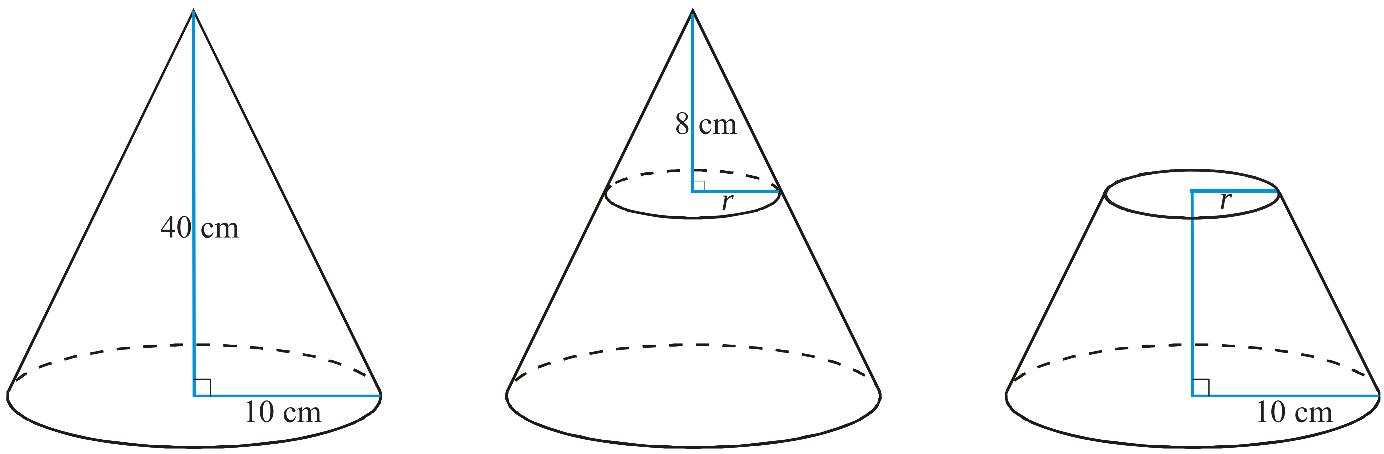
Find h in terms of x.
Give your answer in its simplest form.

Segments and Frustums

1) Find the area of the segment shaded in the diagram below.
Take π to be 3.142.
Give your answer to 3 significant figures.



2) The diagram shows a cone of height 40 cm and base radius 10 cm.
A smaller cone of height 8 cm is removed to form a frustum.



Taking π to be 3.142

a) Work out the radius r of the base of the smaller cone.

Calculate, to the nearest cm^3

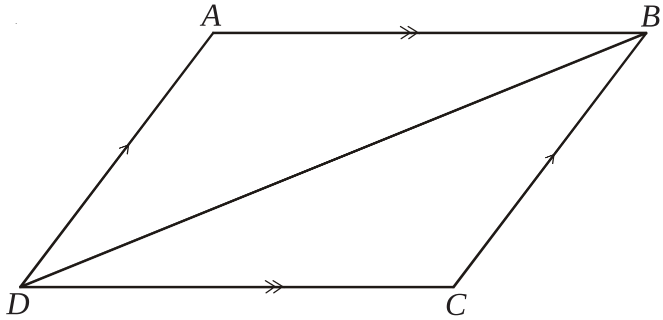
b) The volume of the larger cone.

c) The volume of the smaller cone.

d) The volume of the frustum.

Congruent Triangles

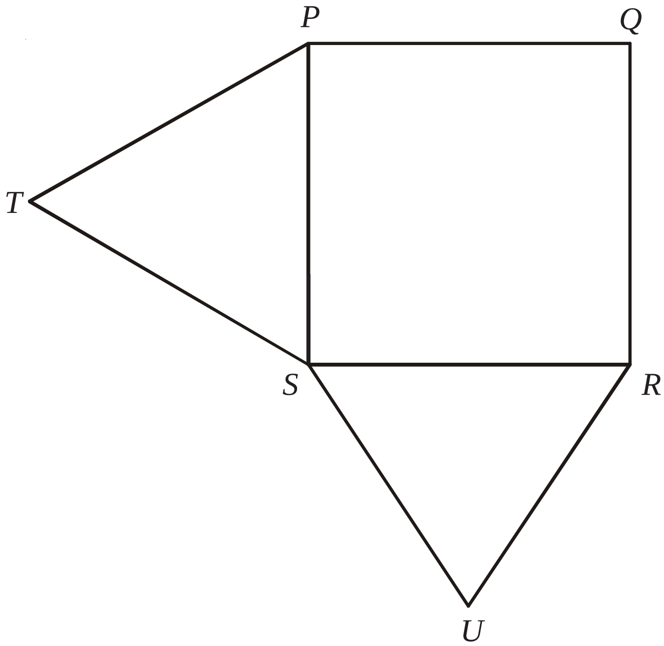
1) *ABCD* is a quadrilateral.



AB is parallel to *DC*.
DA is parallel to *CB*.

Prove that triangle *ABD* is congruent to triangle *CDB*.

2)



PQRS is a square.
PTS and *SUR* are equilateral triangles.

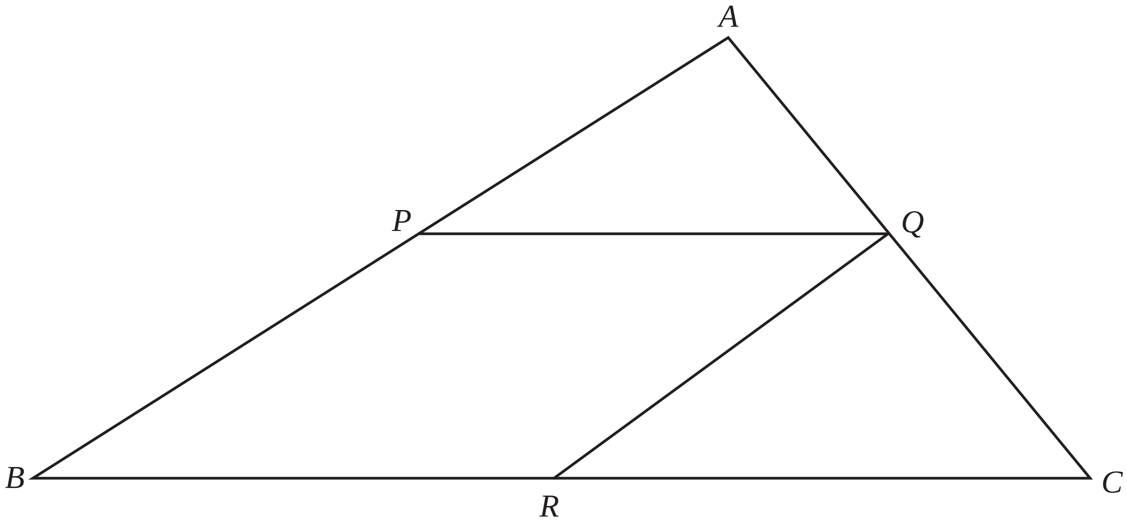
a) Prove that triangle *USP* is congruent to triangle *TSR*.

X is the point such that *RUXT* is a parallelogram.

b) Prove that $UP = UX$

Congruent Triangles

1)

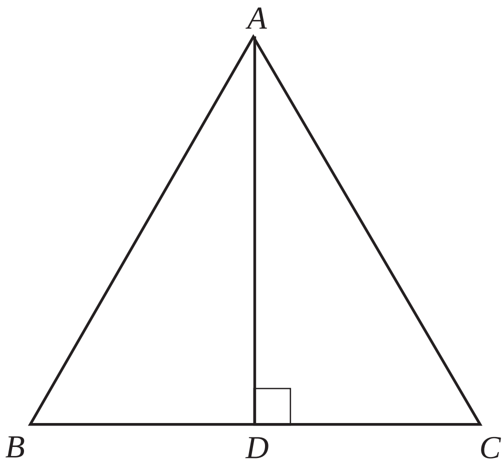


The diagram shows a triangle *ABC*.

PQRB is a parallelogram where
P is the midpoint of *AB*,
Q is the midpoint of *AC*,
and *R* is the midpoint of *BC*.

Prove that triangle *APQ* and triangle *QRC* are congruent.
You must give reasons for each stage of your proof.

2)



ABC is an equilateral triangle.
D lies on *BC*.

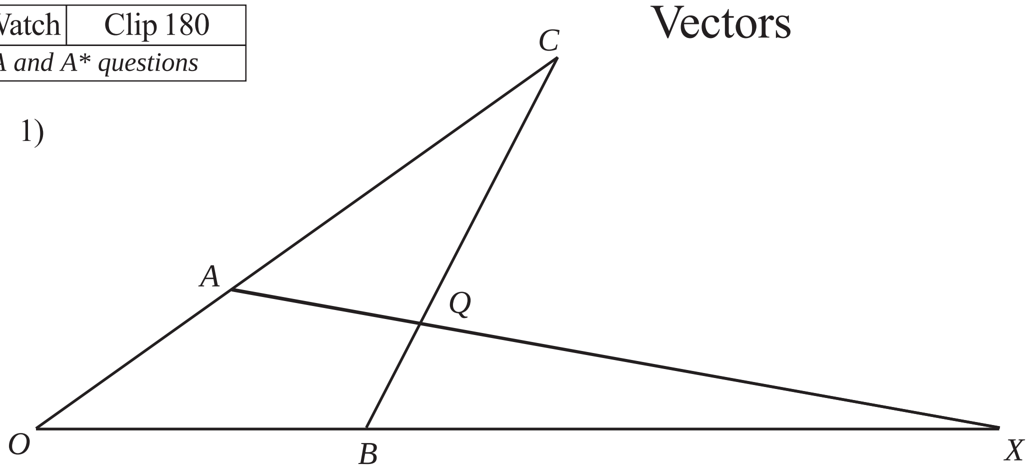
AD is perpendicular to *BC*.

a) Prove that triangle *ADC* is congruent to triangle *ADB*.

b) Hence, prove that $BD = \frac{1}{2}AB$

Vectors

1)



In the diagram,

$\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OB} = 4\mathbf{b}$

OAC, *OBX* and *BQC* are all straight lines.

$AC = 2OA$ and $BQ : QC = 1 : 3$

a) Find, in terms of **a** and **b**, the vectors which represent

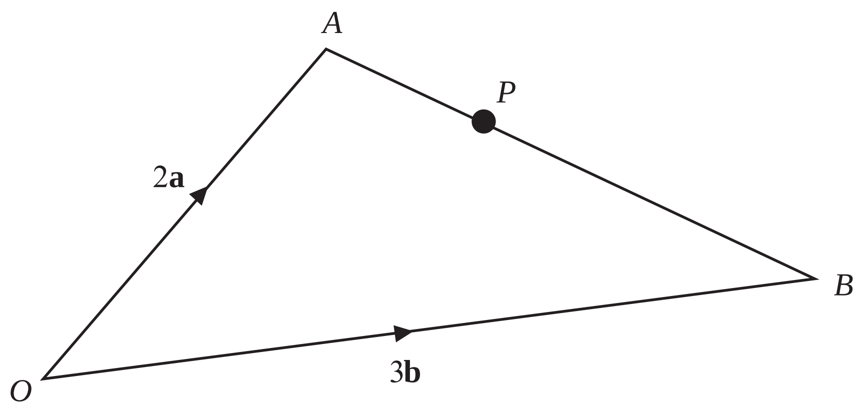
(i) $\overrightarrow{BC}$

(ii) $\overrightarrow{AQ}$

Given that $\overrightarrow{BX} = 8\mathbf{b}$

b) Show that *AQX* is a straight line.

2)



OAB is a triangle.

$\overrightarrow{OA} = 2\mathbf{a}$

$\overrightarrow{OB} = 3\mathbf{b}$

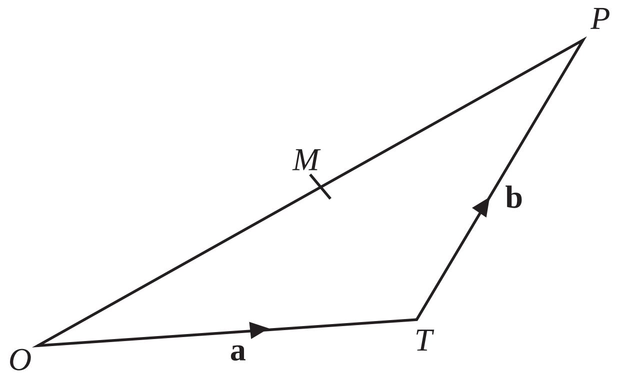
a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

P is a point on *AB* such that $AP : PB = 2 : 3$

b) Show that $\overrightarrow{OP}$ is parallel to the vector $\mathbf{a} + \mathbf{b}$.

Vectors

1)



OPT is a triangle.
M is the midpoint of *OP*.

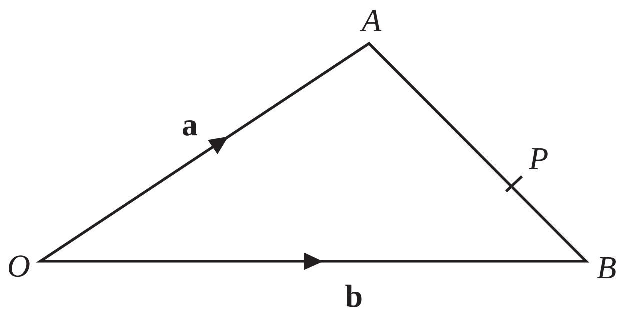
$\overrightarrow{OT} = \mathbf{a}$

$\overrightarrow{TP} = \mathbf{b}$

a) Express $\overrightarrow{OM}$ in terms of **a** and **b**.

b) Express $\overrightarrow{TM}$ in terms of **a** and **b**.
Give your answer in its simplest form.

2)



OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$

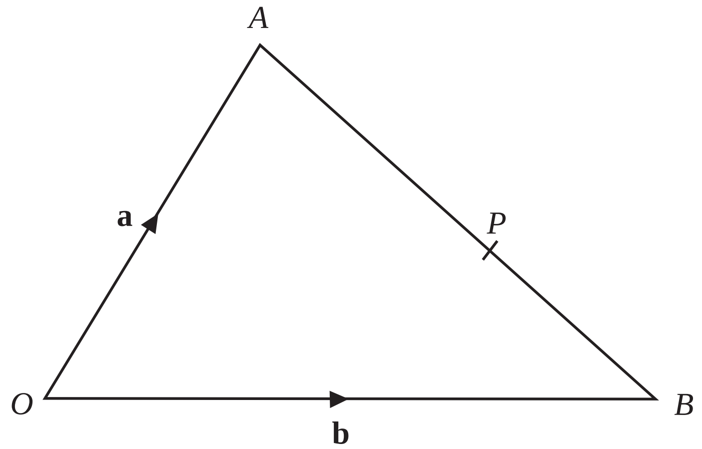
a) Find the vector $\overrightarrow{AB}$ in terms of **a** and **b**.

P is the point on *AB* so that $AP : PB = 2 : 1$

b) Find the vector $\overrightarrow{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

Vectors

1)



OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$

a) Find the vector AB in terms of **a** and **b**.

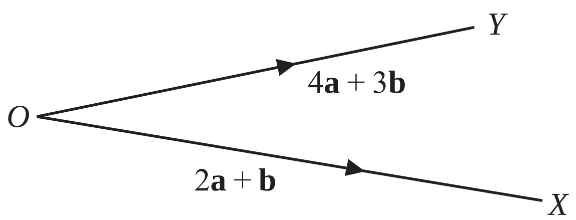
P is the point on AB so that $AP : PB = 3 : 2$

b) Show that $\overrightarrow{OP} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

2)

$OX = 2\mathbf{a} + \mathbf{b}$

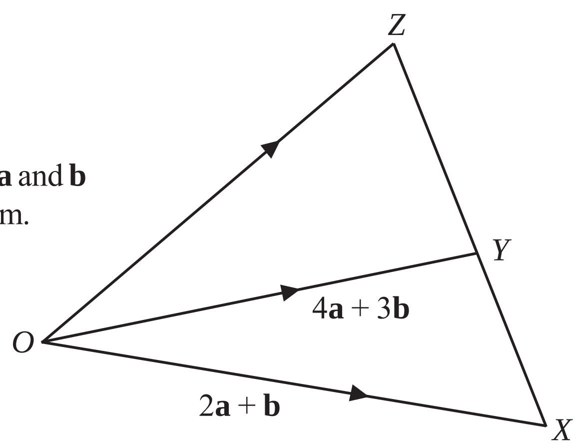
$OY = 4\mathbf{a} + 3\mathbf{b}$



a) Express the vector XY in terms of **a** and **b**
Give your answer in its simplest form.

XYZ is a straight line.
$XY : YZ = 2 : 3$

b) Express the vector OZ in terms of **a** and **b**
Give your answer in its simplest form.



Vectors

1) The diagram shows a trapezium $PQRS$.
$\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{QR} = \mathbf{b}$.
PS is three times the length of QR.

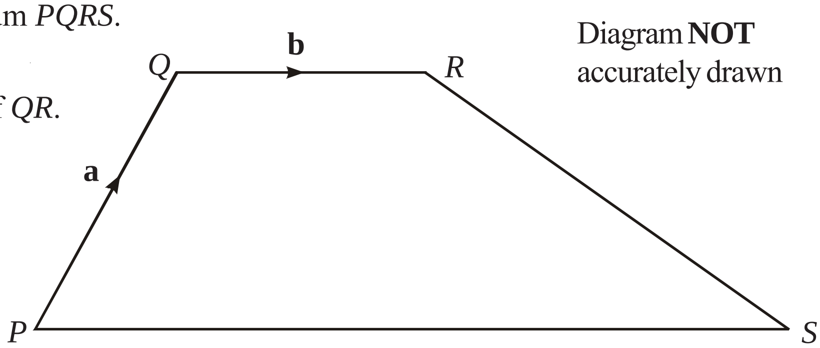


Diagram **NOT** accurately drawn

Find, in terms of **a** and **b**, expressions for

a) $\overrightarrow{QP}$ b) $\overrightarrow{PR}$ c) $\overrightarrow{PS}$ d) $\overrightarrow{QS}$

2) In triangle ABC, P and Q are the midpoints of AB and AC.

$\overrightarrow{AP} = \mathbf{p}$ and $\overrightarrow{AQ} = \mathbf{q}$.

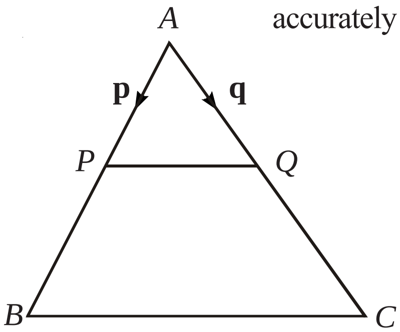


Diagram **NOT** accurately drawn

a) Find, in terms of **p** and **q**, expressions for
(i) $\overrightarrow{PQ}$ (ii) $\overrightarrow{AB}$ (iii) $\overrightarrow{AC}$ (iv) $\overrightarrow{BC}$

b) Use your results from (a) to prove that PQ is parallel to BC.

3)

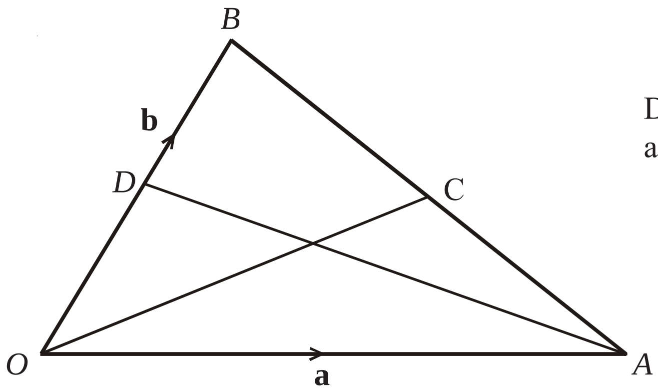


Diagram **NOT** accurately drawn

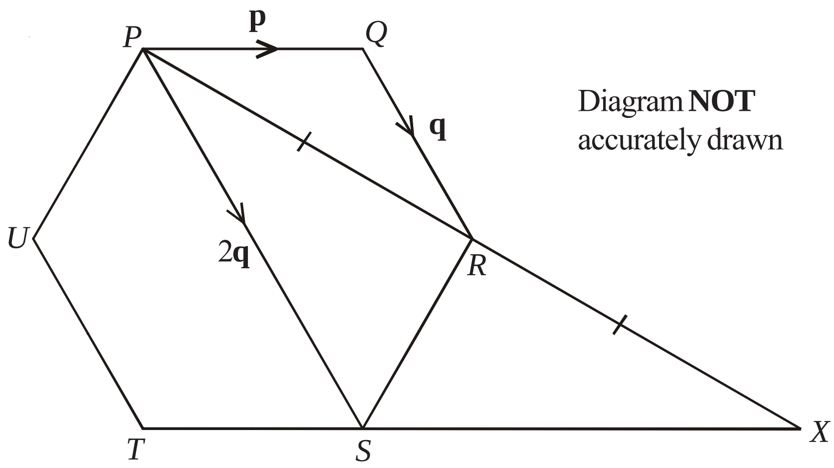
OAB is a triangle.
D is the midpoint of OB.
C is the midpoint of AB.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(i) Find $\overrightarrow{OC}$ in terms of **a** and **b**.

(ii) Show that DC is parallel to OA.

Vectors

1)



PQRSTU is a regular hexagon.

$\overrightarrow{PQ} = \mathbf{p}$ $\overrightarrow{QR} = \mathbf{q}$ $\overrightarrow{PS} = 2\mathbf{q}$

a) Find the vector *PR* in terms of **p** and **q**.

$\overrightarrow{PR} = \overrightarrow{RX}$

b) Prove that *PQ* is parallel to *SX*

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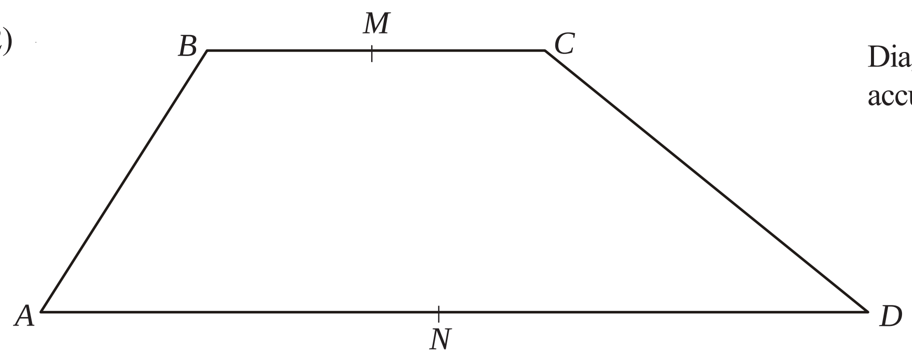
2)



ABCD is a trapezium with *BC* parallel to *AD*.

$\overrightarrow{AB} = 3\mathbf{b}$ $\overrightarrow{BC} = 3\mathbf{a}$ $\overrightarrow{AD} = 9\mathbf{a}$

M is the midpoint of *BC* and *N* is the midpoint of *AD*.

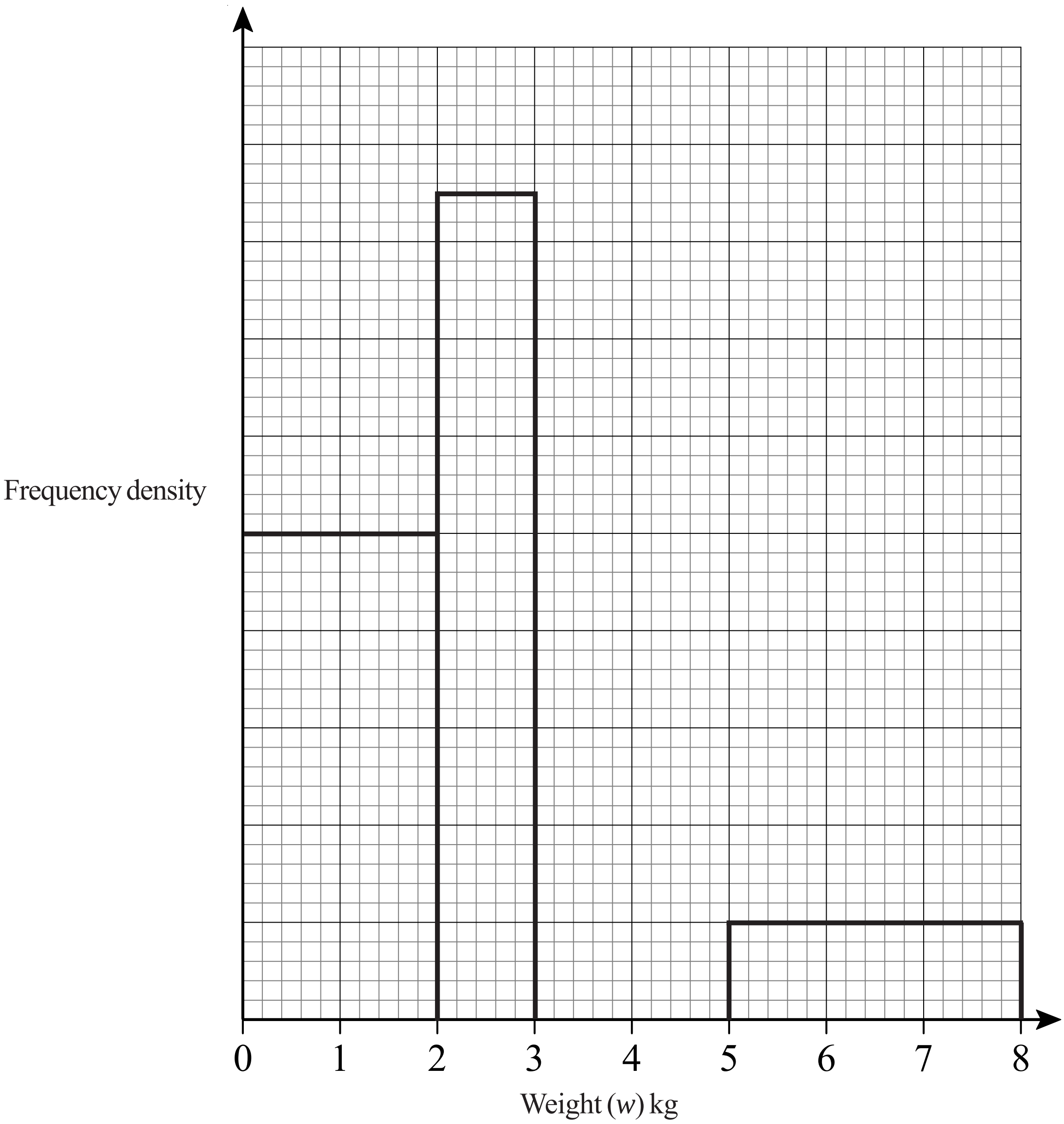
a) Find the vector *MN* in terms of **a** and **b**.

X is the midpoint of *MN* and *Y* is the midpoint of *CD*.

b) Prove that *XY* is parallel to *AD*.

Histograms

1) The table and histogram give some information about the weights of parcels received at a post office during one Thursday.



a) Use the histogram to complete the frequency table.

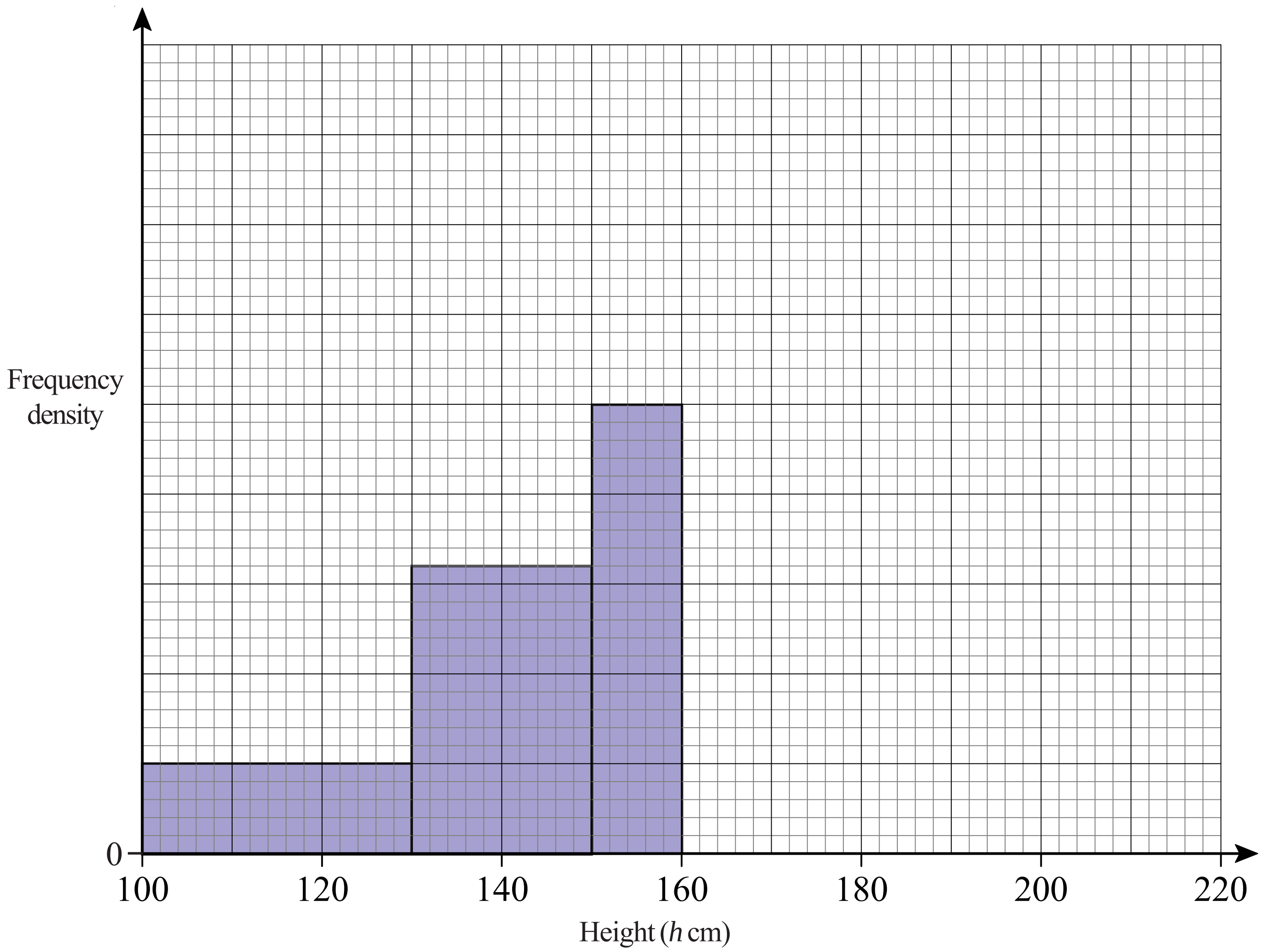
Weight (w) kg	Frequency
$0 < w \leqslant 2$	40
$2 < w \leqslant 3$	
$3 < w \leqslant 4$	24
$4 < w \leqslant 5$	18
$5 < w \leqslant 8$	

b) Use the table to complete the histogram.

Histograms

1) The incomplete table and histogram give some information about the heights (in cm) of some plants.

Height (h cm)	Frequency
$100 < h \leqslant 130$	30
$130 < h \leqslant 150$	
$150 < h \leqslant 160$	
$160 < h \leqslant 180$	40
$180 < h \leqslant 210$	18



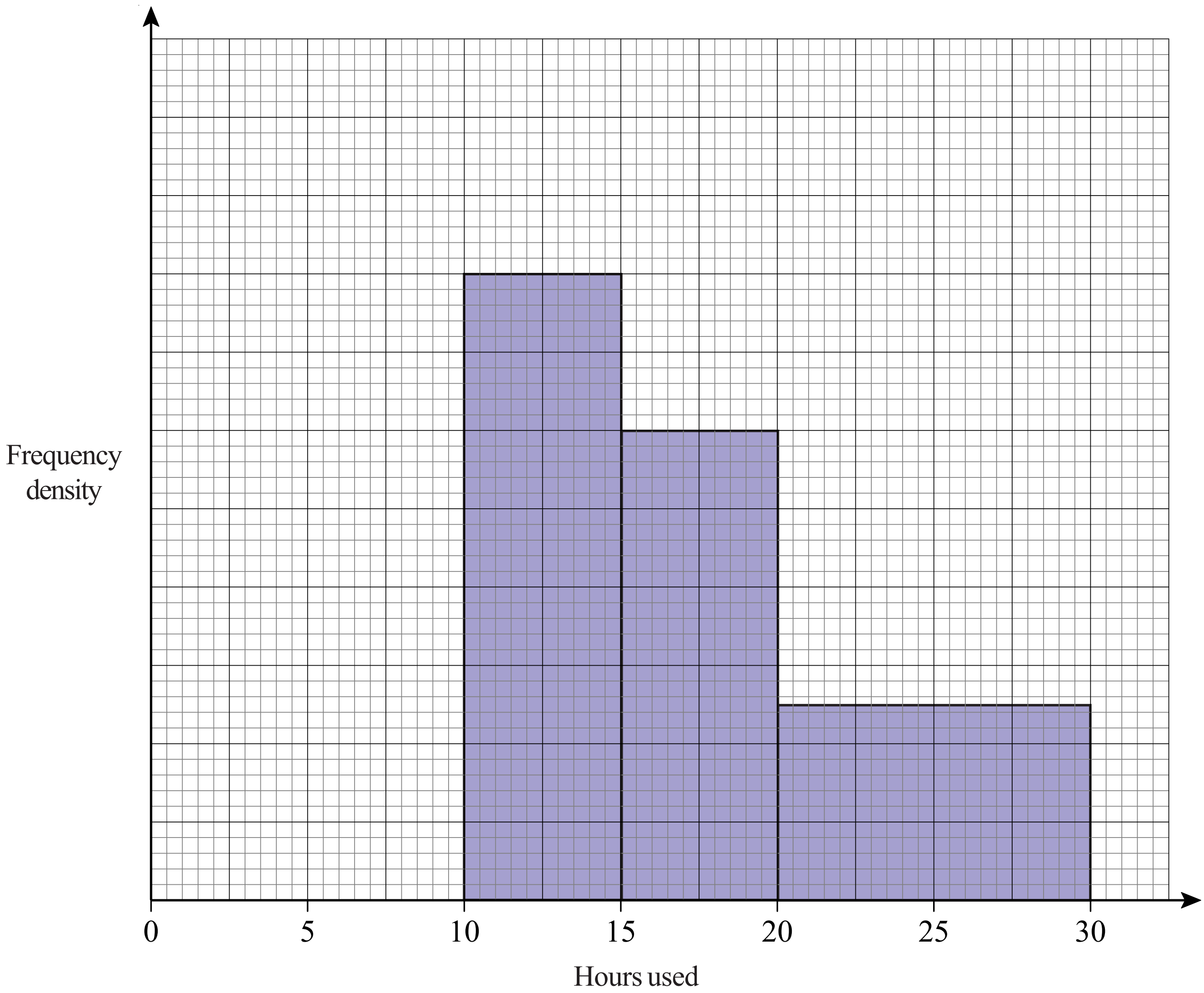
a) Use the histogram to complete the table.

b) Use the table to complete the histogram.

Histograms

1) Paul asked the students in his class how many hours they used the internet for last week.

The incomplete histogram was drawn using his results.



Eight students used the internet for between 10 and 15 hours.
Six students used it for between 0 and 10 hours.

a) Use this information to complete the histogram.

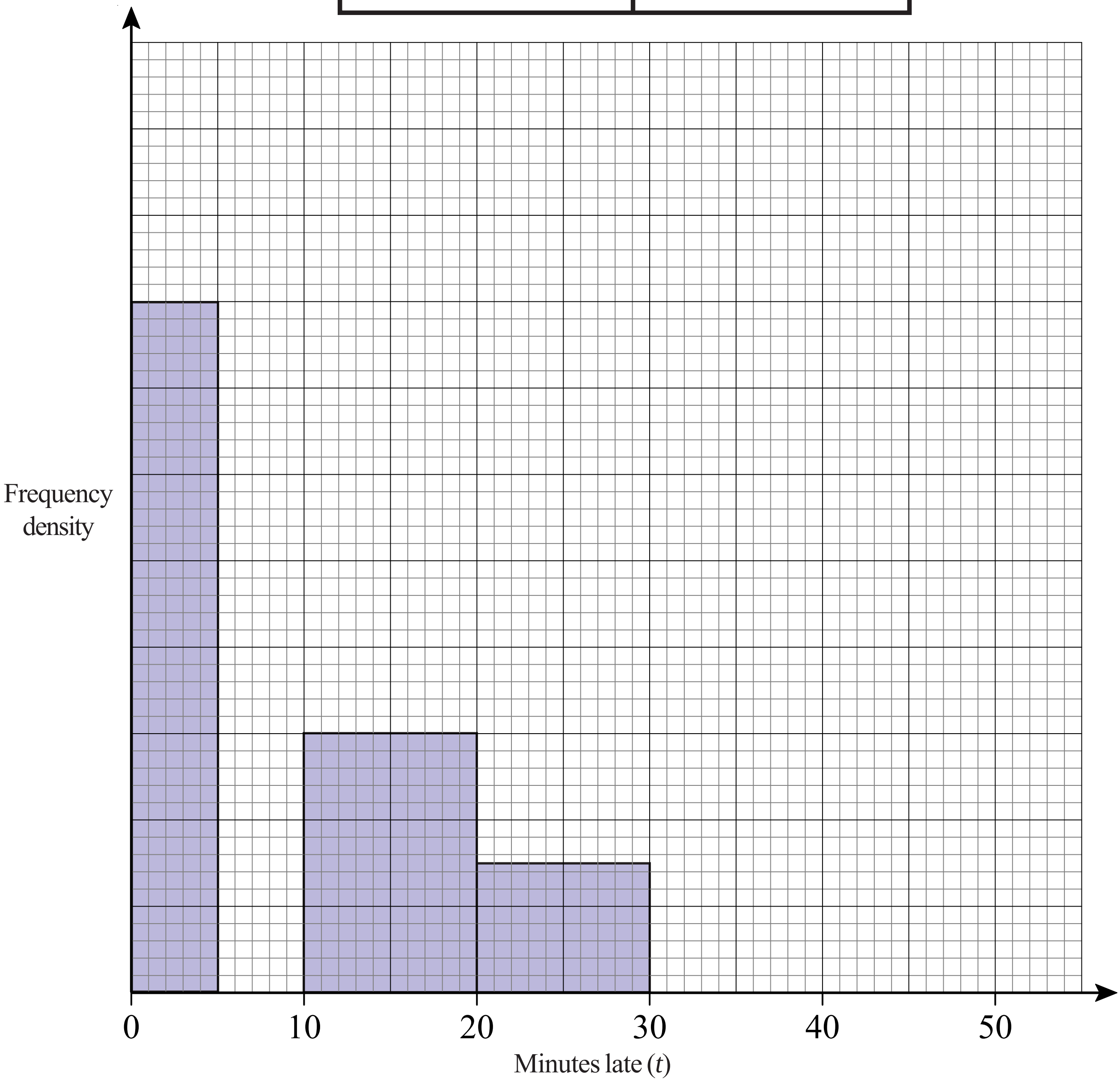
No students used the internet for more than 30 hours.

b) Work out how many students Paul asked.

Histograms

1) Some trains from Nottingham to Leeds were late.
The incomplete table and histogram give some information about how late the trains were.

Minutes late (t)	Frequency
$0 < t \leqslant 5$	16
$5 < t \leqslant 10$	10
$10 < t \leqslant 20$	
$20 < t \leqslant 30$	
$30 < t \leqslant 50$	8



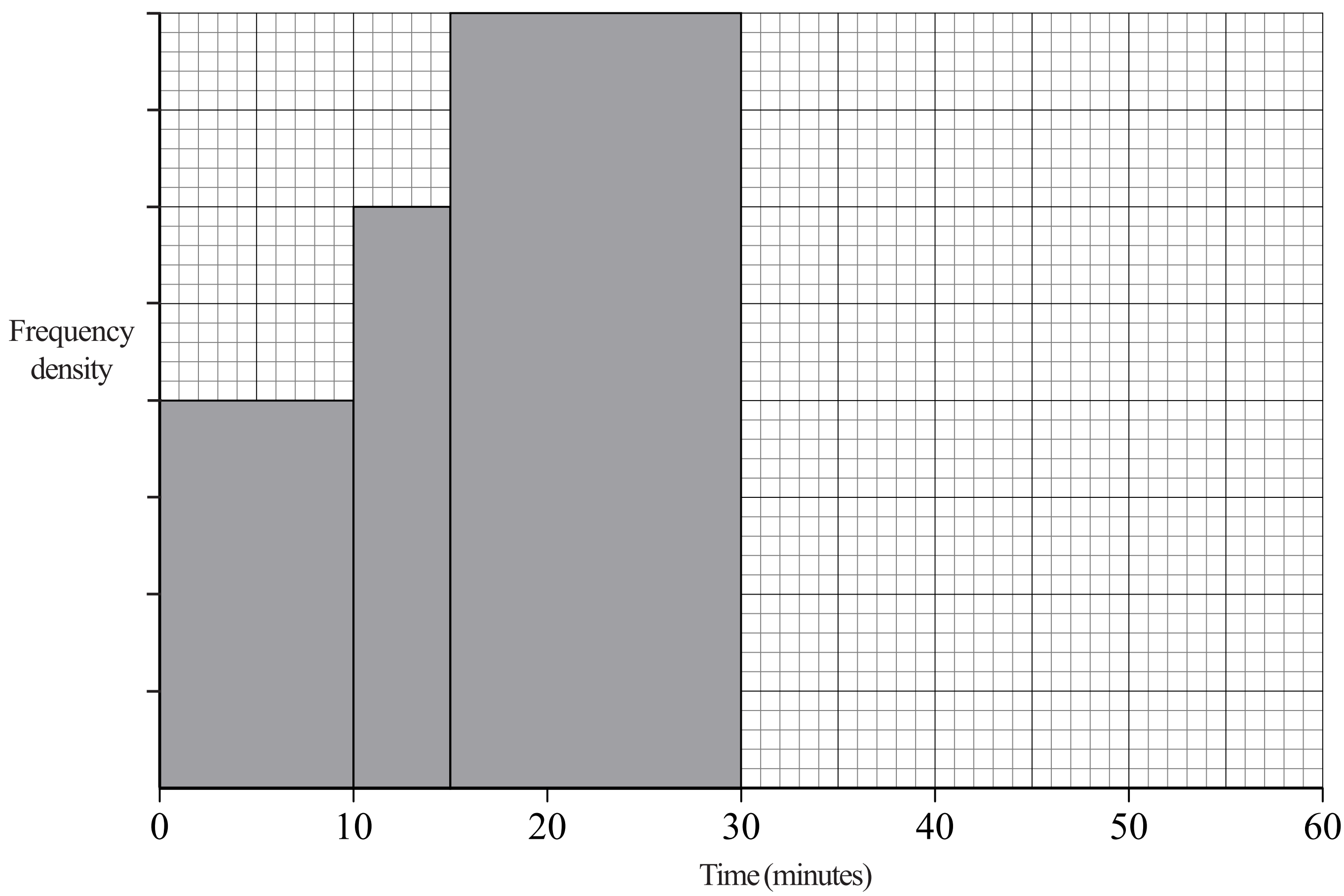
a) Use the information in the histogram to complete the table.

b) Use the information in the table to complete the histogram.

Histograms

1) The table and histogram give information about how long, in minutes, some students took to complete a set of homework.

Time (t) in minutes	Frequency
$0 < t \leqslant 10$	20
$10 < t \leqslant 15$	
$15 < t \leqslant 30$	
$30 < t \leqslant 50$	62
$50 < t \leqslant 60$	23

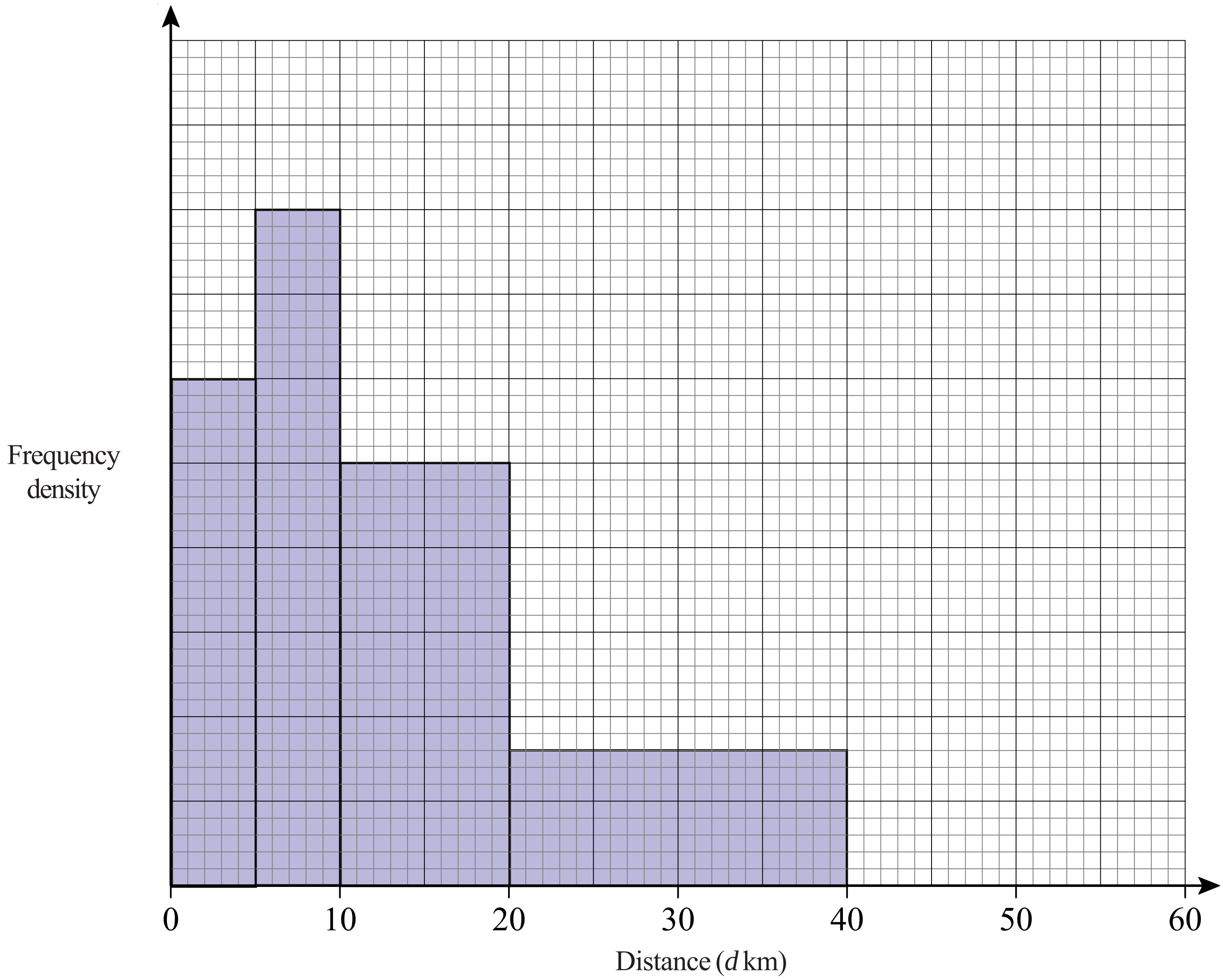


a) Use the information in the histogram to complete the table.

b) Use the table to complete the histogram.

Histograms

1) The incomplete histogram and table give some information about the distances some students travel to school.



a) Use the information in the histogram to complete the frequency table.

Distance (d km)	Frequency
$0 < d \leqslant 5$	15
$5 < d \leqslant 10$	20
$10 < d \leqslant 20$	
$20 < d \leqslant 40$	
$40 < d \leqslant 60$	10

b) Use the information in the table to complete the histogram.

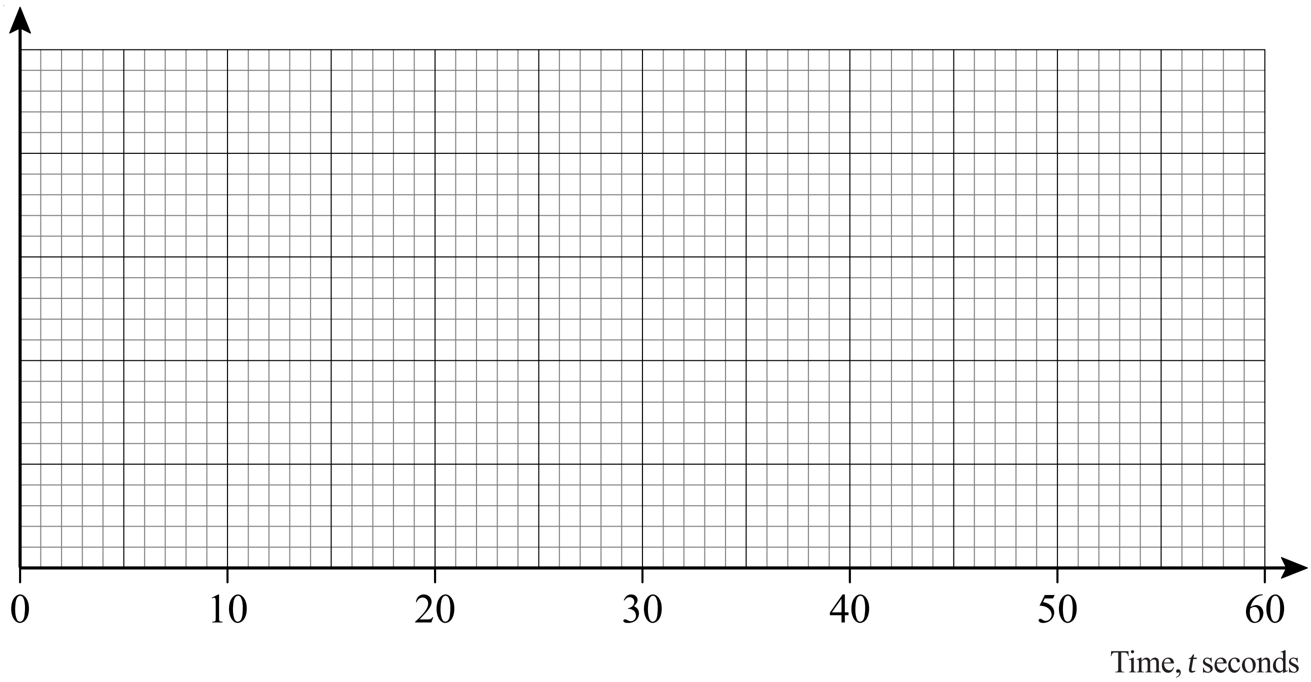
Histograms

1) There are 100 pupils in Year 11. The time taken by each pupil to answer a question was recorded.
The following grouped frequency distribution was obtained.

Time, t seconds	$0 < t \leqslant 10$	$10 < t \leqslant 20$	$20 < t \leqslant 30$	$30 < t \leqslant 40$	$40 < t \leqslant 60$
Number of pupils	6	19	25	36	14

Draw a histogram to illustrate the distribution on the graph paper below.

Time taken to answer in seconds



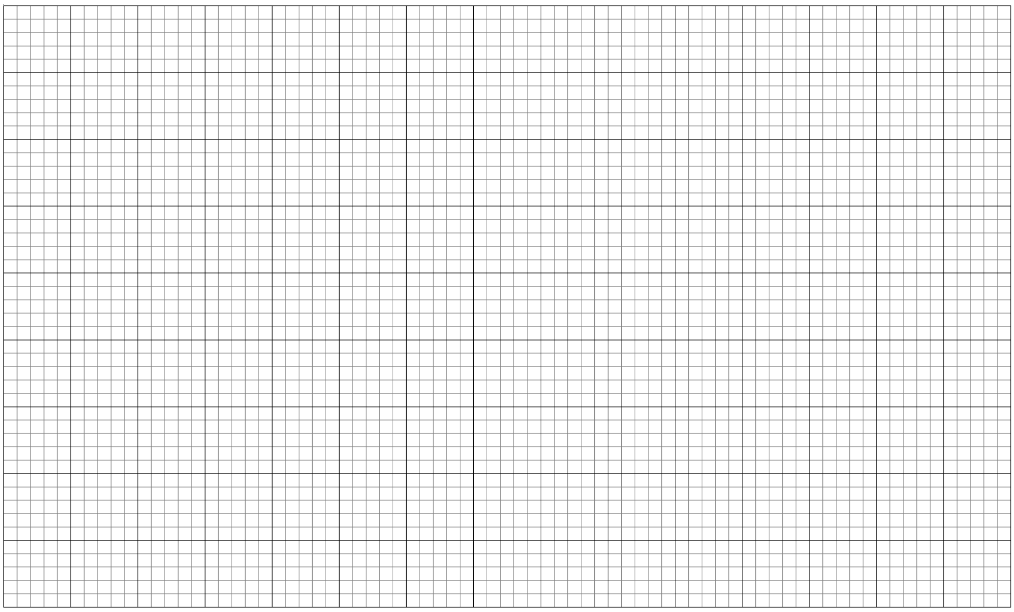
© MathsWatch	Clip 181
Grade A and A questions*	

Histograms

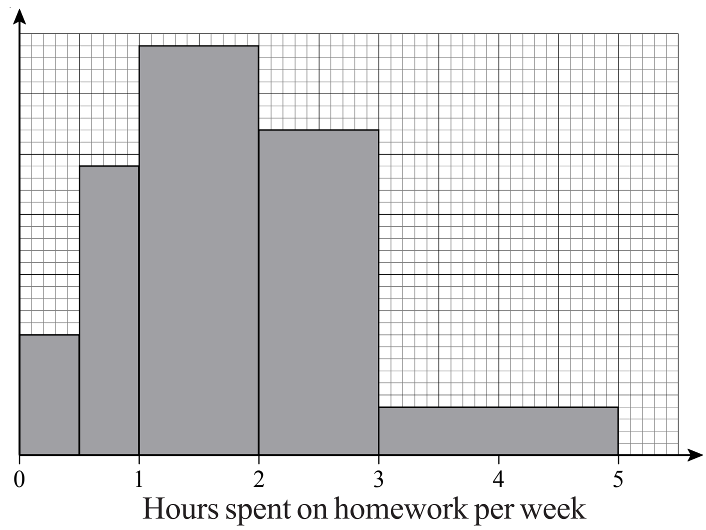
1) The table gives information about the heights, in centimetres, of some 18 year old students.

Height (h cm)	Frequency
$135 < h \leqslant 145$	12
$145 < h \leqslant 165$	46
$165 < h \leqslant 180$	45
$180 < h \leqslant 190$	25
$190 < h \leqslant 195$	4

Use the table to draw a histogram.



2) The histogram shows the amount of time, in hours, that students spend on their homework per week.



Use the histogram to complete the table.

Time (t hours)	Frequency
$0 < t \leqslant ½$	
$½ < t \leqslant 1$	
$1 < t \leqslant 2$	
$2 < t \leqslant 3$	27
$3 < t \leqslant 5$	

Probability ‘And’ & ’Or’ Questions

1) Jordan designs a game for a school fair.
He has two 8-sided spinners.
The spinners are equally likely to land on each of their sides.

One spinner has 3 blue sides, 2 yellow sides and 3 white sides.
The other spinner has 2 blue sides, 2 green sides and 4 white sides.

Calculate the probability that the two spinners will land on the same colour.

2) The probability that it will snow in Paris on Christmas day is 0.06.

a) Work out the probability that it will snow in Paris on **both** Christmas day 2015 **and** Christmas day 2016.

b) Work out the probability that it will snow in Paris on **either** Christmas Day 2015 **or** Christmas Day 2016, but **not** on both.

3) A bag contains 2 black beads, 5 yellow beads and 3 red beads.
Natalie takes a bead at random from the bag, records its colour and replaces it.
She does this two more times.

Work out the probability that, of the three beads Natalie takes, exactly two are the same colour.

Stratified Sampling

1) Ellen wants to do a survey with Years 9, 10 and 11 at her school.
The table shows the number of students in each of these year groups.

Year 11	**Year 10**	**Year 9**
750	700	900

Ellen takes a sample of 50 students stratified by year group.

Work out the number of students from Year 10 in the sample.

2) The table shows information about the year groups of 1000 students in a school.

Year group	7	8	9	10	11	12	13
Number in year	157	180	166	140	132	114	111

Tony takes a sample of 50 of these students, stratified by year group.

Calculate the number of Year 8 students he should have in his sample.

3) The table shows information about Ben's collection of 652 coins.

Country	France	Spain	Germany	Italy	Total
Number of coins	240	182	133	97	652

Ben takes a sample of 50 coins stratified by country.

Work out the number of coins from Italy in this sample.

4)

	Male	**Female**
Lower sixth	399	602
Upper sixth	252	198

The table gives information about the number of students in the two years of a sixth form.

Amy wants to interview some of these students.
She takes a random sample of 70 students stratified by year and by gender.

Work out the number of students in the sample who are male and in the lower sixth.

Stratified Sampling

1) The table below shows the number of employees in each section of a company.

Department	**Managerial**	**Sales**	**Technical**	**Production**
Number of employees	18	45	288	549

A survey on job satisfaction is to be carried out.

a) Explain why a simple random sample of employees is unsuitable.

b) A stratified random sample of 100 is used. Complete the table below to show how many employees from each department will be included.

Department	**Managerial**	**Sales**	**Technical**	**Production**
Number of employees in sample				

2) MathsWatch High-School has 798 pupils.
The size of each year group is shown below.

Year Group	**Boys**	**Girls**
7	77	72
8	74	79
9	72	74
10	93	107
11	85	65

The headteacher wants to find out the opinions of the pupils on changing the timing of the school day. A stratified sample of 80 pupils is taken.

a) Complete the table below to show the numbers of pupils to be sampled.

Year Group	**Boys in Sample**	**Girls in Sample**
7		
8		
9		
10		
11		

The table below shows the number of pupils in the sample who answered YES to a change in the timing of the school day.

Year Group	**Boys in Sample who answered YES**	**Girls in Sample who answered YES**
7	2	3
8	3	5
9	2	1
10	1	4
11	0	1

b) Use the table to estimate the percentage of pupils in the school who would answer YES to the question.